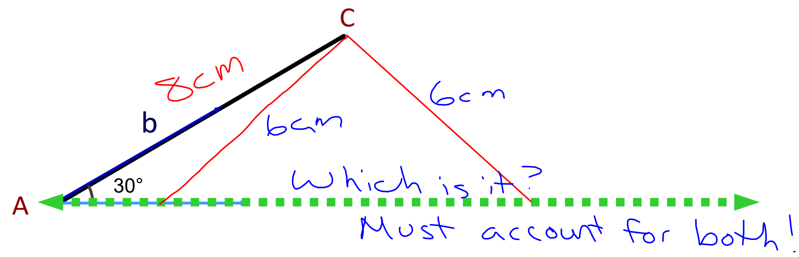
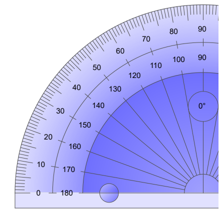
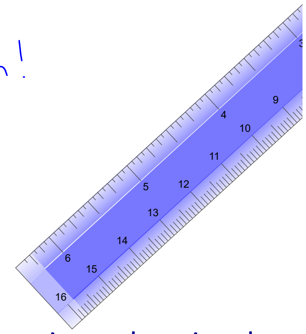
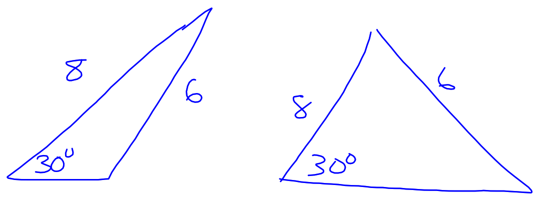
Lesson 4.4B: Sine Law - AMBIGUOUS Case

Draw triangle Δ ABC, a = 6 cm, b = 8 cm, A = 30°.



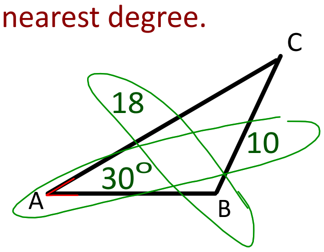






- When two sides and the non-included angle of a triangle are given, the triangle may not be unique. (SSA)
- You will have to determine if there is no solution, one solution or two possible solutions.

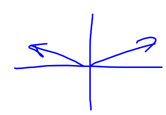
Ex. 1: Given that ΔABC has <A = 30°, a = 10, and b = 18, find the value of <B to the nearest degree.



$$\frac{\sin B}{18} = \frac{\sin 30}{10}$$

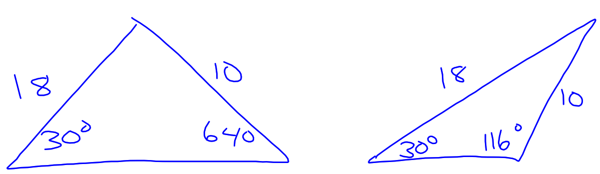
$$\sin B = 18 \cdot \frac{\sin 30}{10}$$

$$B \doteq 64°$$



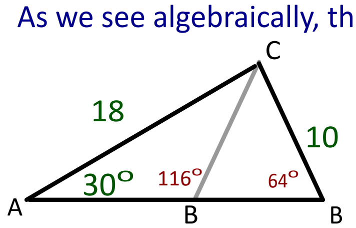
Q1
$\theta = 64°$

Q2
$\theta = 180 - 64$
$= 116°$



$\therefore \angle B = 64°, 116°$

As we see algebraically, there are two possible answers to this question.

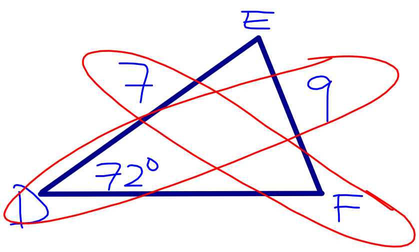


NOTE: Angles are supplementary (add to 180°)

Therefore, it is very important to always *consider* both solutions (Q1 & Q2) when using Sine Law to solve a triangle given SSA.

Ex. 2: Determine the measures of all angles in the given triangles.

a) In ΔDEF, $\angle D = 72^{\circ}$, d = 9 cm, f = 7 cm.



SSA! Need to account for ambiguous case.

$$\frac{\sin F}{7} = \frac{\sin 72}{9}$$

$$\sin F = 7 \cdot \frac{\sin 72}{9}$$

$$F \doteq 47.7^{\circ}$$

Q1
$\theta = 47.7^{\circ}$

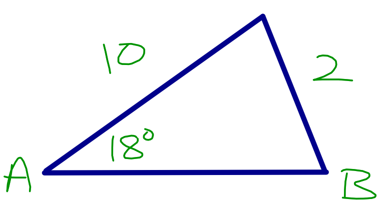
Q2
$\theta = 180 - 47.7$
$= 132.3^{\circ}$

Triangle 1
$\angle E = 180 - 72 - 47.7^{\circ}$
$= 60.3^{\circ}$ VALID

Triangle 2
$\angle E = 180 - 72 - 132.3^{\circ}$
$= -24.3^{\circ}$ INVALID

$\therefore$ Only 1 solution
$\angle E = 60.3^{\circ}$
$\angle F = 47.7^{\circ}$
$\angle D = 72^{\circ}$

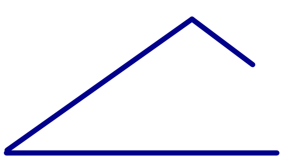
b) In ΔABC, $\angle A = 18^{\circ}$, a = 2 cm, b = 10 cm.



$$\frac{\sin B}{10} = \frac{\sin 18}{2}$$

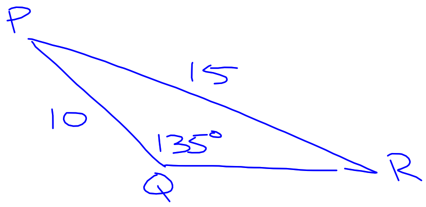
B = ERROR

$\therefore$ NO SOLUTIONS!



This example shows the case of no solution.
The triangle cannot be constructed as side "a" is too short.

c) In ΔPQR, < Q = 135°, q = 15 cm, r = 10cm.



$$\frac{\sin R}{10} = \frac{\sin 135}{15}$$

$R \doteq 28.1°$

SSA!

Q_1/ $\theta = 28.1°$

Q_2/ $\theta = 180 - 28.1° = 151.9°$

Both valid?

Triangle 1 $\angle P = 180 - 135 - 28.1 = 16.9°$ VALID

Triangle 2 $\angle P = 180 - 135 - 151.9$ = INVALID

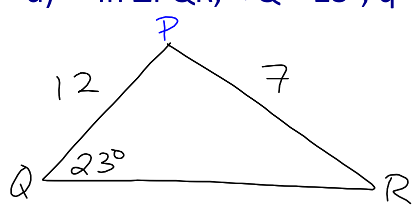
∴ Only 1 solution

$\angle P = 16.9°$

$\angle R = 28.1°$

$\angle Q = 135°$

d) In ΔPQR, < Q = 23°, q = 7cm, r = 12cm.



$$\frac{\sin R}{12} = \frac{\sin 23}{7}$$

$R \doteq 42.1°$

Q_1 $\theta = 42.1°$

Q_2 $\theta = 137.9°$

SSA!

Triangle 1

$\angle P = 180 - 23 - 42.1° = 114.9$ VALID

Triangle 2

$\angle P = 180 - 23 - 137.9° = 19.1°$ VALID

Pull

∴ Two Solutions

$\angle P = 114.9°$	$\angle P = 19.1°$
$\angle R = 42.1°$	$\angle R = 137.9°$
$\angle Q = 23°$	$\angle Q = 23°$

Homework
pg. 318 #2, 3, 4a, 5, 6

am•big•u•ous

doubtful,uncertain,
unclear in meaning