

7.1 Simple and Compound Interest

Simple Interest: Interest is earned only on the original investment.

Simple Interest Formulas:

$$I = Prt \qquad A = P + I$$

where

A = Amount at the end of investment (\$)
 P = Principal or original amount (\$)
 r = Rate of interest per year (decimal)
 t = Time invested (years)
 I = Total interest earned (\$)

Ex. 1 Veeta invests \$900 at 5%/a for 7 years.

- a) How much interest does she earn?
- b) What is the total amount in the account?

a) $I = Prt$
 $P = 900$
 $r = 0.05$
 $t = 7$
 $I = 900(0.05)(7) = 315$
 ∴ She earned \$315

b) $A = P + I$
 $= 900 + 315$
 $= 1215$
 ∴ She has \$1215

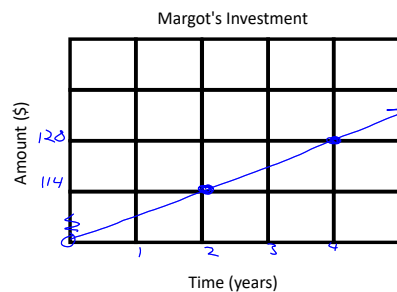
Ex. 2 Margot invests \$100 at 7%/a for 5 years.

- a) Complete the table to examine what happens to her investment.

Year	Interest (\$)	Amount (\$)
0		100
1	7	107
2	7	114
3	7	121
4	7	128
5	7	135

What type of sequence does this represent?
 Arithmetic!

- b) Sketch the growth of her money over the 5 years.



What type of growth does this represent?
 LINEAR!

Interest is constant: \$ 7 / yr
 Slope is 7

Simple Interest:

- Increases by the same amount of money for each time interval.
- Creates an arithmetic sequence.
- Represents linear growth.

Compound Interest:

- Interest is added to the principal for the next compound period.
- Has the effect of paying/earning interest on interest.

Ex. 1 Consider Margot's investment of \$100 at 7% if the interest is compounded yearly.

a) Complete the table to examine what happens to her investment.

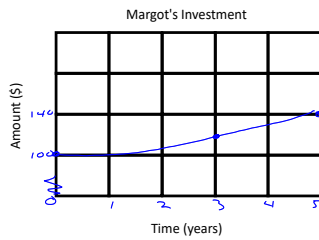
Year	Interest (\$)	Amount (\$)
0		100
1	7	107
2	7.49	114.49
3	8.01	122.50
4	8.58	131.08
5	9.18	140.26

7% of 100 = 7
7% of 107 = 7.49
7% of 114.49 = 8.01
7% of 122.50 = 8.58
7% of 131.08 = 9.18

x 1.07
x 1.07

What type of sequence does this represent?
Geometric!

b) Sketch the growth of her money over the 5 years.



What type of growth does this represent?

Amount has a constant ratio
 $r = 1.07$

Compound Interest:

- Increases by a constant multiplier for each compound period.
- Creates a geometric sequence.
- Represents exponential growth.

Compound Interest Formulas:

$A = P(1 + i)^n$ $I = A - P$

This is the formula for exponential growth. The growth factor is $(1 + i)$.

where
 P = Principal or amount invested/borrowed (\$)
 A = Amount at the end of the investment (\$)
 i = Interest rate per compound period (decimal)

$i = \frac{\text{rate}}{\text{\# of compounding periods per year}}$

n = Number of compound periods

$n = (\text{\# of years}) \times (\text{\# of periods per year})$

Compounding Periods --> How often interest is compounded.

Typical compound periods	# of compounds / year
annually	1
semi-annually	2
quarterly	4
monthly	12
bi-monthly	24
weekly	52
bi-weekly	26
daily	365

Twice a month ←

"semi" = half
"bi" = two

Ex. 2 Myla invests \$1500 in an account paying 4.75%/a compounded quarterly. How much money will she have at the end of 5 years?

$$P = 1500$$

$$A = ?$$

$$i = \frac{0.0475}{4}$$

$$n = 5 \text{ yrs} \times 4/\text{yr}$$

$$= 20$$

$$A = P(1+i)^n$$

$$= 1500(1 + \frac{0.0475}{4})^{20}$$

$$= 1899.45$$

∴ She will have \$1899.45

Ex. 3 Sarah needs to borrow \$4500 to buy her first car.
(She will not be making payments but will pay it off in one lump sum in 5 years.)

She has 2 options:

- a) 3.4%/a for 5 years compounded monthly OR
- b) 3.9%/a for 5 years compounded semi-annually.

Which option is better?

a)

$$P = 4500$$

$$A = ?$$

$$i = \frac{0.034}{12}$$

$$n = 5 \cdot 12 = 60$$

$$A = 4500(1 + \frac{0.034}{12})^{60}$$

$$= 5337.59$$

↑
Less to pay back!

∴ Option A is better

b)

$$P = 4500$$

$$A = ?$$

$$i = \frac{0.039}{2}$$

$$n = 2 \cdot 5 = 10$$

$$A = 4500(1 + \frac{0.039}{2})^{10}$$

$$= 5458.64$$

Ex. 4 Don has \$24 000 invested in a University fund that he hopes will grow to \$30 000 in 3 years. What interest rate, compounded quarterly will he need to invest at in order to achieve his goal?

$$P = 24000$$

$$A = 30000$$

$$i = \frac{r}{4}$$

$$n = 3 \cdot 4 = 12$$

$$30000 = 24000(1 + \frac{r}{4})^{12}$$

$$\frac{30000}{24000} = (1 + \frac{r}{4})^{12}$$

$$\sqrt[12]{1.25} = 1 + \frac{r}{4}$$

$$0.01877 = \frac{r}{4}$$

$$r = 0.075$$

∴ The rate would need to be 7.5%

HOMEWORK
Pg. 481 # 3, 4, 6
Pg. 490 # 1, 6, 7, 9

