# STATION A

1. The point (5,-7) lies on the terminal arm of angle $\theta$ . Determine the value of:
   a) $\cos\theta$ b) $\cot\theta$

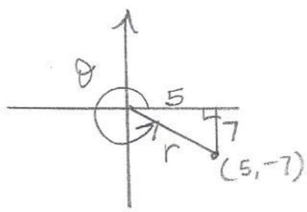
2. If $\sec\theta = \frac{-8}{3}$, $180^\circ \le \theta \le 270^\circ$ , determine the value of $\theta$.

3. Determine the exact simplified value of the following. Show all your work.

$$(\sin(-45^\circ))^2(\cos 210^\circ)^2 - (\cos 180^\circ)(\cot 315^\circ)$$

## Solutions

1.




$r = \sqrt{5^2 + (-7)^2}$

$r = \sqrt{74}$

a) $\cos\theta = \frac{x}{r}$

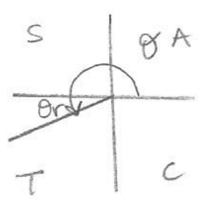
$= \frac{5}{\sqrt{74}}$

b) $\cot\theta = \frac{x}{y}$

$= \frac{5}{-7}$

$= -\frac{5}{7}$

2.




$\sec\theta = -\frac{8}{3}$

$\therefore \cos\theta = -\frac{3}{8}$

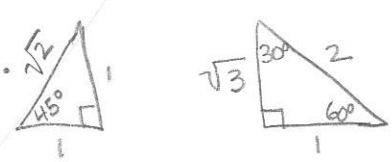
$\theta_r = \cos^{-1}\left(\frac{3}{8}\right)$

$\theta_r \doteq 68^\circ$

$\therefore \theta = 180^\circ + 68^\circ$

$\theta = 248^\circ$

3.




$\sin(-45^\circ)$

$= -\sin 45^\circ$

$= \frac{-1}{\sqrt{2}}$

S A, T C, -45°

$\theta_r = 45^\circ$

$\cos 210^\circ$

$= -\cos 30^\circ$

$= \frac{-\sqrt{3}}{2}$

S 210° A, T C

$\theta_r = 30^\circ$

$\cos 180^\circ$

$= -1$

(-1,0)

$\cot 315^\circ$

$= -\cot 45^\circ$

$= -1$

S 315° A, T C

$\theta_r = 45^\circ$

$\therefore (\sin(-45^\circ))^2(\cos 210^\circ)^2 - (\cos 180^\circ)(\cot 315^\circ)$

$= \left(\frac{-1}{\sqrt{2}}\right)^2\left(\frac{-\sqrt{3}}{2}\right)^2 - (-1)(-1)$

$= \left(\frac{1}{2}\right)\left(\frac{3}{4}\right) - 1$

$= \frac{3}{8} - 1$

$= \frac{3}{8} - \frac{8}{8}$

$= \frac{-5}{8}$ //

# STATION B

1. Determine the value of the following. Use exact values where possible.
   a) sin 225 b) tan 120 c) cos 160 d) sec 330

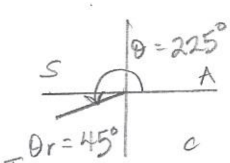
2. State an expression equivalent to:
   a) $\cos^2\theta - 1$ b) $\dfrac{1}{\csc\theta}$ c) $\dfrac{\cos^2\theta}{\sin^2\theta}$

## Solutions

1\. a) $\sin 225^\circ$

$= -\sin 45^\circ$

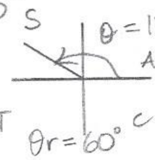
$= -\dfrac{1}{\sqrt{2}}$




b) $\tan 120^\circ$

$= -\tan 60^\circ$

$= -\sqrt{3}$




c) $\cos 160^\circ$

$= -0.9397$

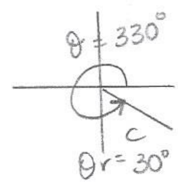
d) $\sec 330^\circ$

$= (\cos 330^\circ)^{-1}$

$= (\cos 30^\circ)^{-1}$

$= \left(\dfrac{\sqrt{3}}{2}\right)^{-1}$

$= \dfrac{2}{\sqrt{3}}$




2\. a) $\cos^2\theta - 1$

$= \sin^2\theta$

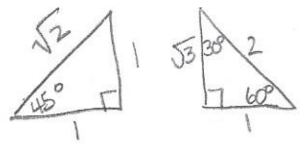
b) $\dfrac{1}{\csc\theta}$

$= \sin\theta$

c) $\dfrac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

# STATION C




1. Determine the value of angle θ, for $0° \le \theta \le 360°$.

a) $\cos\theta = \frac{-\sqrt{2}}{2}$

b) $\cot\theta = \sqrt{3}$

c) $\sin\theta = -1$

d) $\tan\theta = -2.1445$

2. Determine the number of triangles possible. Do NOT solve.

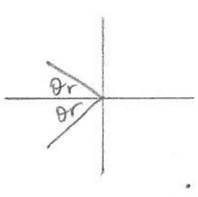
a) $\triangle ABC$, $A = 135°$, $a = 12$cm, $b = 9$cm

b) $\triangle DEF$, $D = 42°$, $d = 5$cm, $e = 7$cm

c) $\triangle GHI$, $G = 72°$, $g = 17$cm, $h = 15$cm

## Solutions

#1. a) $\cos\theta = \frac{-\sqrt{2}}{2}$




$\theta_r = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

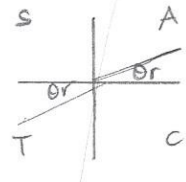
$\theta_r = 45°$ (from Δ)

∴ In Q2: $\theta_p = 180° - 45° = 135°$

In Q3: $\theta_p = 180° + 45° = 225°$

$\therefore \theta = \{135°, 225°\}$

b) $\cot\theta = \sqrt{3}$




$\therefore \tan\theta = \frac{1}{\sqrt{3}}$

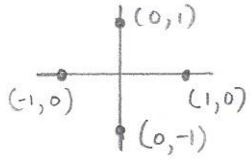
$\theta_r = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $\frac{opp}{adj}$

$\theta_r = 30°$ (from special Δ).

∴ In Q3: $\theta_p = 180° + 30° = 210°$

$\therefore \theta = \{30°, 210°\}$

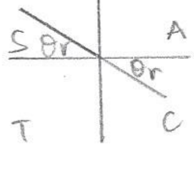
c) $\sin\theta = -1$




$\therefore \theta = \{270°\}$

sin θ is the y-coordinate

d) $\tan\theta = -2.1445$




$\theta_r = \tan^{-1}(2.1445) \doteq 65°$

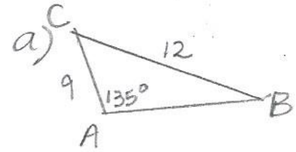
In Q2: $\theta_p = 180° - 65° = 115°$

In Q4: $\theta_p = 360° - 65° = 295°$

$\therefore \theta = \{115°, 295°\}$

## Solutions #2

a)




SSA Ambiguous Case?

$\frac{\sin B}{9} = \frac{\sin 135°}{12}$

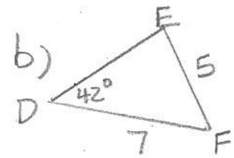
$\angle B = \sin^{-1}\left(\frac{9\sin 135°}{12}\right)$

$\angle B \doteq 32°$ in Q1

or $\angle B = 148°$ in Q2

$\because 135° + 148° > 180°$, ∴ there is only one solution.

b)




SSA

$\frac{\sin E}{7} = \frac{\sin 42°}{5}$

$\angle E \doteq 70°$ in Q1

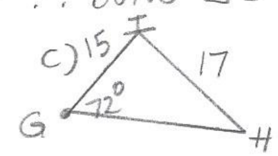
or $\angle E = 110°$ in Q2

$\because 42° + 70° < 180°$

& $\because 42° + 110° < 180°$,

∴ both Δ's are possible

c)




SSA

$\frac{\sin H}{15} = \frac{\sin 72°}{17}$

$\angle H \doteq 57°$ in Q1

or $\angle H = 123°$ in Q2

$\because 72° + 123° > 180°$, ∴ there is only 1 triangle.

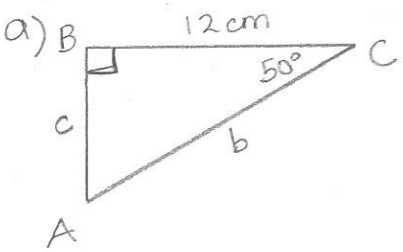
# STATION D

Solve the following triangles. Include a diagram as part of your solution.

a) $\triangle ABC, B = 90°, C = 50°, a = 12cm$

b) $\triangle DEF, D = 110°, e = 17.2cm, f = 5.9cm$

a)




① $\angle A = 90° - 50°$ (CAT)

$\angle A = 40°$

② $\tan 50° = \frac{c}{12}$

$\therefore c = 12 \tan 50°$

$c \doteq 14.3$ cm

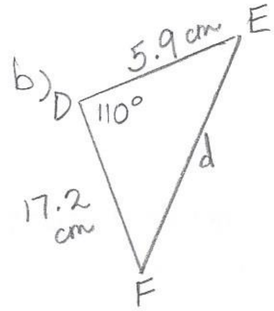
③ $\cos 50° = \frac{12}{b}$

$\therefore b = \frac{12}{\cos 50°}$

$b \doteq 18.7$ cm

or use P.T.

b)




Given SAS use Cosine Law.

① $d^2 = 17.2^2 + 5.9^2 - 2(17.2)(5.9)\cos 110°$

$d \doteq 20.0$ cm

② Use Sine Law to find a 2nd angle.

$\frac{\sin E}{17.2} = \frac{\sin 110°}{20.0}$

$\angle E = \sin^{-1}\left(\frac{17.2 \sin 110°}{20.0}\right)$

$\angle E \doteq 54°$

③ By SATT,

$\angle F = 180° - 110° - 54°$

$\angle F = 16°$

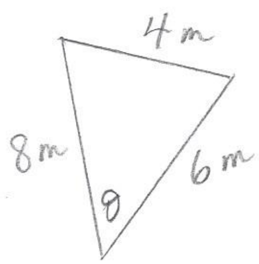
# STATION E

1. Determine the value of the measure of the smallest angle in a triangle with sides of 4m, 6m, and 8m. Include a diagram as part of your solution.

2. Julia enters a road race that starts on Cavanagh Side Road. Runners leave point A and run for 8km at an angle of 24° to Cavanagh Side Road to reach checkpoint B. At checkpoint B runners turn and run 4 km to the finish line at point C which is also located on Cavanagh Side Road. When Julia finishes, how far is she from where she started the race?

Solutions

1.




The smallest angle is across from the shortest side.

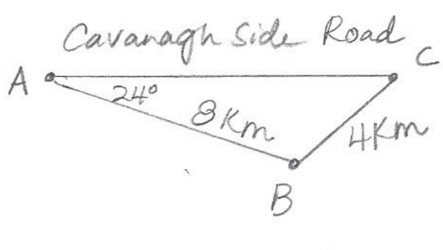
Given SSS, use Cosine Law.

$\theta = \cos^{-1}\left(\frac{6^2+8^2-4^2}{2(6)(8)}\right)$

$\theta = \cos^{-1}(0.875)$

$\theta \doteq 29°$.

2.




Wanted: Length of AC.
Given: SSA
Must consider ambiguous case, using Sine Law.

① $\frac{\sin C}{8} = \frac{\sin 24°}{4}$

$\angle C = \sin^{-1}\left(\frac{8\sin 24°}{4}\right)$

$\angle C \doteq 54°$ in Q1

or $\angle C = 126°$ in Q2.

$\because 24° + 54° < 180°$

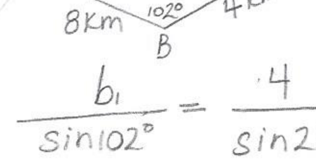
& $24° + 126° < 180°$,

$\therefore$ there are 2 solutions.

② If $\angle C = 54°$,

$\angle B = 180° - 24° - 54°$ (SATT)

$\angle B = 102°$




$\frac{b_1}{\sin 102°} = \frac{4}{\sin 24°}$

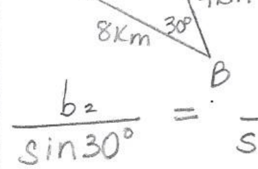
$b_1 = \frac{4\sin 102°}{\sin 24°}$

$b_1 \doteq 9.6$

③ If $\angle C = 126°$,

$\angle B = 180° - 24° - 126°$ (SATT)

$\angle B = 30°$




$\frac{b_2}{\sin 30°} = \frac{4}{\sin 24°}$

$b_2 = \frac{4\sin 30°}{\sin 24°}$

$b_2 \doteq 4.9$

$\therefore$ Julia is either 4.9Km or 9.6 Km from where she started.

# STATION F

Prove the following identities.

a) $\tan x + \cot x = \csc x \sec x$

b) $\dfrac{\sin^2 x}{1-\cos x} = 1+\cos x$

## Solutions

a) $LS = \tan x + \cot x$ $\qquad RS = \csc x \sec x$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\cos x \sin x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$= \sec x \csc x$$

$\because LS = RS$

$\therefore$ Q.E.D.

b) $LS = \dfrac{\sin^2 x}{1-\cos x}$ $\qquad RS = 1+\cos x$

$$= \frac{1-\cos^2 x}{1-\cos x}$$

$$= \frac{(1-\cancel{\cos x})(1+\cos x)}{(1-\cancel{\cos x})}$$

$$= 1+\cos x$$

$\because LS = RS$

$\therefore$ Q.E.D.