1.3 Factoring

$$3(x+2) \underset{\text{Factor}}{\overset{\text{Expand/Multiply}}{\rightleftarrows}} 3x+6$$

To factor is to write an algebraic expression as a product of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:
- Solve equations
- Graph relations

In grade 10 you learned how to:
- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring

Always your first and last step.

2 or more terms

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn - 4mnt$

$= 2mn(1 - 2t)$

b) $6t^5 - 9t^2$

$= 3t^2(2t^3 - 3)$

c) $3x^4 - 6x^3 + 9x$

$= 3x(x^3 - 2x^2 + 3)$

d) $4x(a-b) - 3(a-b)$

$= (a-b)(4x-3)$

Factor by Grouping

An even # of terms: 4, 6, 8, etc...

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

The terms *m - 5* and *5 - m* are opposites. This means that one divided by the other is *-1*.

a) $3x(m-5)+2(5-m)$

$= 3x(m-5)+2(-1)(m-5)$

$= 3x(m-5)-2(m-5)$

$= (m-5)(3x-2)$

b) $x(y-2)-4(2-y)$

$= x(y-2)+4(y-2)$

$= (y-2)(x+4)$

c) $mx+2y+my+2x$

$= mx+2x+my+2y$

$= x(m+2)+y(m+2)$

$= (m+2)(x+y)$

d) $22vx-6vy+11wx-3wy$

$= 22vx+11wx-6vy-3wy$

$= 11x(2v+w)-3y(2v+w)$

$= (2v+w)(11x-3y)$

e) y^2+1-y^3-y

$= 1(y^2+1)-y(y^2+1)$

$= (y^2+1)(1-y)$

OR

$= -y^3+y^2-y+1$

$= -y^2(y-1)-1(y-1)$

$= (y-1)(-y^2-1)$

$= (-1)(y-1)(y^2+1)$

$= (1-y)(y^2+1)$

f) $16x^5+8x^4-6x^3-3x^2+4x+2$

$= 8x^4(2x+1)-3x^2(2x+1)+2(2x+1)$

$= (2x+1)(8x^4-3x^2+2)$

Simple Trinomials

3 terms

$ax^2 + bx + c$ where $a = 1$

$(x + n_1)(x + n_2)$

M = ac

A = b

N = n_1, n_2

a) $x^2 - 9x + 14$

$= (x-7)(x-2)$

M 14

A -9

N -7 -2

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$

$= 5(x+7)(x-4)$

M -28

A 3

N 7 -4

c) $a^2 + 8ab + 15b^2$

$= (a+3b)(a+5b)$

M 15

A 8

N 3 5

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 - 4b)(x^2 + 6b)$

$x^2 + 2xb - 24b^2$

$= (x-4b)(x+6b)$

M -24

A 2

-4 6

Difference of Squares

2 terms

2 perfect squares separated by a subtraction: $a^2 - b^2$

$a^2 - b^2 = (a - b)(a + b)$

conjugates

a) $49x^2 - 16y^2$

$= (7x - 4y)(7x + 4y)$

b) $3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x - 2)(x + 2)$

c) $a^2 - \frac{1}{9}$

$= (a - \frac{1}{3})(a + \frac{1}{3})$

d) $81 - m^{12}$

$= (9 - m^6)(9 + m^6)$

$= (3 - m^3)(3 + m^3)(9 + m^6)$

e) $(3x-2)^2 - (5x+1)^2$