1.3 Factoring

$$3(x+2) \underset{\text{Factor}}{\overset{\text{Expand/Multiply}}{\rightleftarrows}} 3x+6$$

To factor is to write an algebraic expression as a product of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:

- Solve equations
- Graph relations

In grade 10 you learned how to:

- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring

Always your first and last step.

2 or more terms

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn-4mnt$

$= 2mn(1-2t)$

b) $6t^5-9t^2$

$= 3t^2(2t^3-3)$

c) $3x^4-6x^3+9x$

$= 3x(x^3-2x^2+3)$

d) $4x\underline{(a-b)}-3\underline{(a-b)}$

$= (a-b)(4x-3)$

Factor by Grouping

An even # of terms: 4, 6, 8, etc...

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

! The terms $m - 5$ and $5 - m$ are opposites. This means that one divided by the other is -1.

a) $3x(m-5)+2(5-m)$

$= 3x(m-5)+2(-1)(m-5)$

$= 3x(m-5)-2(m-5)$

$= (m-5)(3x-2)$

b) $x(y-2)-4(2-y)$

$= x(y-2)+4(y-2)$

$= (y-2)(x+4)$

c) $mx+2y+my+2x$

$= mx+2x+my+2y$

$= x(m+2)+y(m+2)$

$= (m+2)(x+y)$

d) $22vx-6vy+11wx-3wy$

$= 22vx+11wx-6vy-3wy$

$= 11x(2v+w)-3y(2v+w)$

$= (2v+w)(11x-3y)$

e) y^2+1-y^3-y

$= 1(y^2+1)-y(y^2+1)$

$= (y^2+1)(1-y)$

OR

$= -y^3+y^2-y+1$

$= -y^2(y-1)-1(y-1)$

$= (y-1)(-y^2-1)$

$= (-1)(y-1)(y^2+1)$

$= (1-y)(y^2+1)$

f) $16x^5+8x^4-6x^3-3x^2+4x+2$

$= 8x^4(2x+1)-3x^2(2x+1)+2(2x+1)$

$= (2x+1)(8x^4-3x^2+2)$

Simple Trinomials

3 terms

$ax^2 + bx + c$ where $a = 1$

$(x + n_1)(x + n_2)$

M = ac

A = b

N = n_1, n_2

a) $x^2 - 9x + 14$

$= (x-7)(x-2)$

M 14

A -9

N -7 -2

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$

$= 5(x+7)(x-4)$

M -28

A 3

N 7 -4

c) $a^2 + 8ab + 15b^2$

$= (a+3b)(a+5b)$

M 15

A 8

N 3 5

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 - 4b)(x^2 + 6b)$

$x^2 + 2xb - 24b^2$

$= (x-4b)(x+6b)$

M -24

A 2

-4 6

Difference of Squares

WHEN?

2 terms

2 perfect squares separated by a subtraction: $a^2 - b^2$

HOW?

$a^2 - b^2 = (a - b)(a + b)$

conjugates

a) $49x^2 - 16y^2$

$= (7x - 4y)(7x + 4y)$

b) $3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x - 2)(x + 2)$

c) $a^2 - \frac{1}{9}$

$= \left(a - \frac{1}{3}\right)\left(a + \frac{1}{3}\right)$

d) $81 - m^{12}$

$= (9 - m^6)(9 + m^6)$

$= (3 - m^3)(3 + m^3)(9 + m^6)$

e) $(3x-2)^2 - (5x+1)^2$

$= \big((3x-2) - (5x+1)\big)\big((3x-2) + (5x+1)\big)$

$= (3x - 2 - 5x - 1)(3x - 2 + 5x + 1)$

$= (-2x - 3)(8x - 1)$

f)

$x^4 - 16$

$= (x^2 + 4)(x^2 - 4)$

$= (x^2 + 4)\underbrace{(x + 2)(x - 2)}$

$x^2 \qquad -4$

$x^2 - 4$

$= x^2 + 2x - 2x - 4$

$= x(x + 2) - 2(x + 2)$

$= (x + 2)(x - 2)$

M -4

A 0

N $2, -2$

Complex Trinomials

3 terms

$ax^2 + bx + c$ where $a \neq 1$

$(a_1x + f_1)(a_2x + f_2)$

M = ac
A = b
N = n_1, n_2

1. Use a, n_1 and n_2 to find the factors.
$$\frac{a}{n_1}, \frac{a}{n_2}$$

2. Reduce.
$$\frac{a_1}{f_1}, \frac{a_2}{f_2}$$

OR

Decompose the middle term using n_1, n_2 and factor by grouping.

a) $10x^2 - 11x - 6$

$= 10x^2 - 15x + 4x - 6$
$= 5x(2x-3) + 2(2x-3)$
$= (2x-3)(5x+2)$

M -60
A -11
N -15, 4

a) $10x^2 - 11x - 6$

$= (2x-3)(5x+2)$

M -60
A -11
N -15 4

$\frac{-15}{10}$ $\frac{4}{10}$

$-\frac{3}{2}$ $\frac{2}{5}$

b) $14x^2 + 31xy - 10y^2$

$= (7x-2y)(2x+5y)$

M -140
A 31
N -4 35

$\frac{-4}{14}$ $\frac{35}{14}$

$-\frac{2}{7}$ $\frac{5}{2}$

-140
1 140
2 70
4 35

c) $18a^2b + 3ab - 6b$

$= 3b(6a^2 + a - 2)$
$= 3b(2a-1)(3a+2)$

M -12
A 1
N -3 4

$\frac{-3}{6}$ $\frac{4}{6}$

$-\frac{1}{2}$ $\frac{2}{3}$

d) $3x^4 - 25x^2 - 18$

$= (x^2-9)(3x^2+2)$
$= (x+3)(x-3)(3x^2+2)$

M -54
A -25
N -27 2

$\frac{-27}{3}$ $\frac{2}{3}$

$-\frac{9}{1}$ $\frac{2}{3}$

Perfect Square Trinomials

WHEN?

3 terms

$ax^2 + bx + c$

where a & c are perfect squares and
b is twice the product of their square roots.

HOW?

$$\left(\sqrt{a}x \pm \sqrt{c}\right)^2$$

same sign as b

a) $m^2 + 10m + 25$

$= (m + 5)^2$

b) $2x^2 - 24x + 72$

$= 2(x^2 - 12x + 36)$

$= 2(x - 6)^2$

c) $16a^2 + 24a + 9$

$= (4a + 3)^2$

d) $x^4 - 8x^2 + 16$

$= (x^2 - 4)^2$

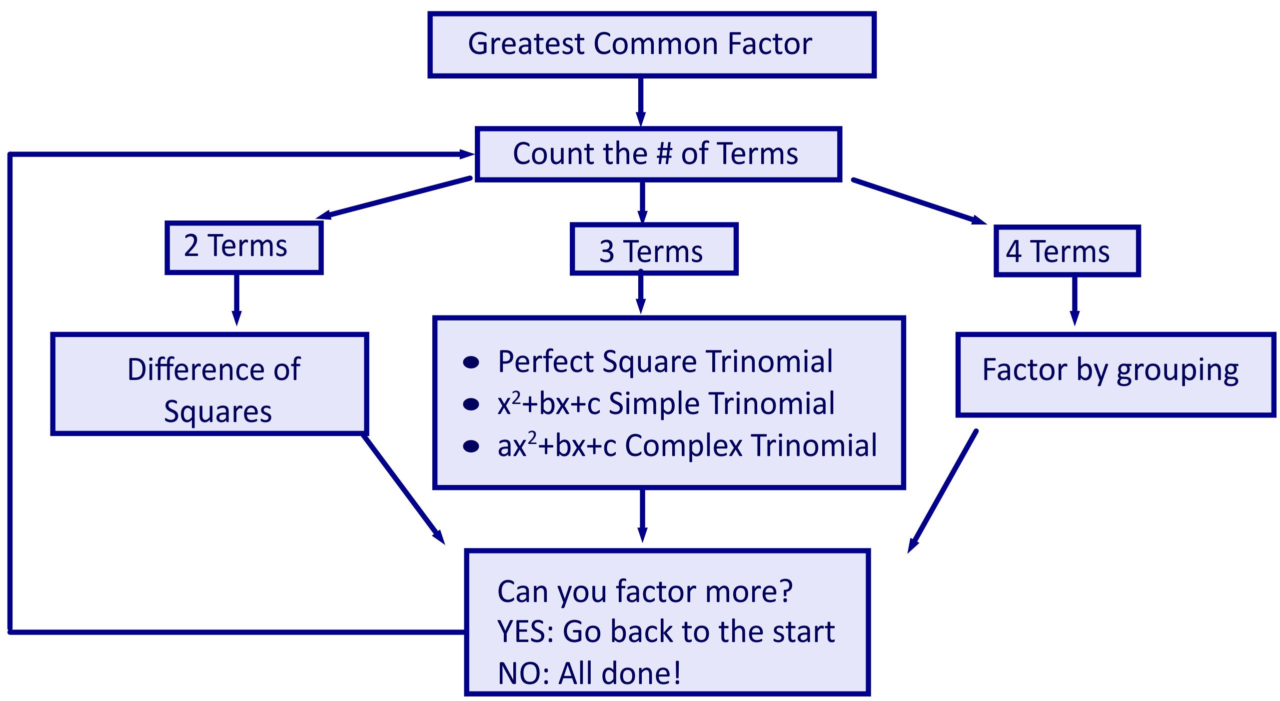
$= [(x+2)(x-2)]^2$

$= (x+2)^2(x-2)^2$

$x^2 - 4$

$= (x+2)(x-2)$

Factoring Flowchart



Check you answer by EXPANDING

HOMEWORK
Handout 1.3