Lesson 1.4A: Completing the Square

~ Max or Min of a Quadratic Function

What do you remember about quadratics?

Standard Form	Vertex Form	Factored Form
$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$	$f(x) = a(x-r)(x-s)$
y-int is "c"	vertex is (h,k)	x-int are "r" and "s"

"a" in all three forms is the same and indicates the direction of opening and the vertical stretch/compression

Completing the Square

Why do we need this process?

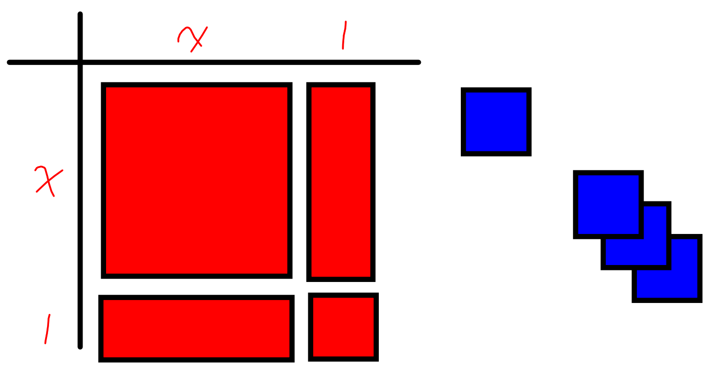
This is the method to convert from STD => vertex

What it is: $f(x) = ax^2 + bx + c$ $\quad$ $f(x) = a(x-h)^2 + k$

What it is not: FACTORING or SOLVING

RECALL: Completing the Square with Algetiles $f(x)=a(x-h)^2+k$

x^2+2x-3



$\therefore \quad x^2+2x-3=(x+1)^2-4$

You are forcing part of the trinomial to be a <u>perfect square</u> which can then be factored to a <u>binomial squared</u>.

$f(x) = ax^2 + bx + c$ $\xrightarrow{\text{complete the square}}$ $f(x) = a(x-h)^2 + k$

$\xleftarrow{\text{expand and simplify}}$

- hard to graph
- vertex unknown

- easy to graph
- vertex & transformations known

What information can you read from vertex form? $f(x) = a(x-h)^2 + k$

- direction of opening and stretch factor 'a'
- the vertex (h,k)
- max/min value & when it occurs

Ex. 1 Find the value of c that makes a perfect square trinomial.

a) $x^2 + 8x + c$ $\left(\frac{b}{2}\right)^2$

$\left(\frac{8}{2}\right)^2 = 16$

$c = 16$

b) $x^2 - 10x + c$

$c = \left(-\frac{10}{2}\right)^2$

$= 25$

c) $x^2 + 3x + c$

$c = \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

$\left\{ \frac{3^2}{2^2} \right.$

Ex. 2 Change to vertex form by completing the square.

a) $f(x) = x^2 - 12x + 8$

$= \underbrace{x^2 - 12x + 36} - 36 + 8$

$= (x-6)^2 - 28$

Process

⇒ Common factor the first two terms so the coefficient of x^2 is 1.

⇒ Add and subtract the number that will make a perfect square trinomial.

⇒ Remove the compensation term from the brackets.

⇒ Factor the perfect square and collect the remaining terms.

b) $f(x) = 2x^2 - 12x + 23$

$= 2(x^2 - 6x + 9 - 9) + 23$

$(-9)(2)$

$= 2(x^2 - 6x + 9) - 18 + 23$

$= 2(x-3)^2 + 5$

Ex. 3 Determine the max/min value and when it occurs.

a) $f(x) = -4x^2 - 5x - 3$

$= -4\left(x^2 + \frac{5}{4}x + \frac{25}{64} - \frac{25}{64}\right) - 3$

$= -4\left(x^2 + \frac{5}{4}x + \frac{25}{64}\right) + \frac{25}{16} - 3$

$\frac{-25}{\cancel{64}_{16}} \times \cancel{-4}^{1}$

$= -4\left(x + \frac{5}{8}\right)^2 - \frac{23}{16}$

$\frac{5}{4} \div 2$

$= \frac{5}{4} \times \frac{1}{2}$

$= \frac{5}{8}$

$\left(\frac{5}{8}\right)^2 = \frac{25}{64}$

$\frac{25}{16} - \frac{48}{16}$

$= -\frac{23}{16}$

b) $f(x) = \frac{2}{3}x^2 + 7x - \frac{1}{2}$

☆ Challenge question ☆

Ex. 4 A football player kicks a ball off a football tee. The height of the ball, h, in metres after t seconds, can be modelled using the formula: $h(t) = -5t^2 + 20t$. What is the maximum height of the ball?

std => vertex
complete the square

y of vertex

$$h(t) = -5t^2 + 20t$$
$$= -5(t^2 - 4t + 4 - 4)$$
$$\hookrightarrow (-4)(-5)$$
$$= -5(t^2 - 4t + 4) + 20$$
$$= -5(t-2)^2 + 20$$

Vertex $(2, 20)$

$\therefore$ Max height of 20m
(at $t = 2s$)

Homework

p. 153 #1, 4 (only complete the square), 8

I just completed this.
I think math ~~guys~~ people will love it.