

## 1.4B Partial Factoring

### ~Max or Min of a Quadratic Function

Recall that to find the vertex we have:

- completed the square (time consuming)
- factored (vertex falls halfway between the zeros) (gr.10 and later in the unit)

And now for something sort of brand new...

### Finding the Vertex by Partial Factoring

Partial Factoring involves finding two points on the parabola that have the same y-coordinate.

$$f(x) = 3x^2 - 24x + 3$$

What is the y-intercept?

it is the "c" value from standard form

$$(0, 3)$$

is there another x-value with the same y-value?

$$\text{- need to have } 3x^2 - 24x = 0$$

Partially Factored Form:

$$f(x) = 3x(x - 8) + 3$$

$$(0, 3) \quad (8, 3)$$

Now, determine the the axis of symmetry and the vertex.

Axis of symmetry

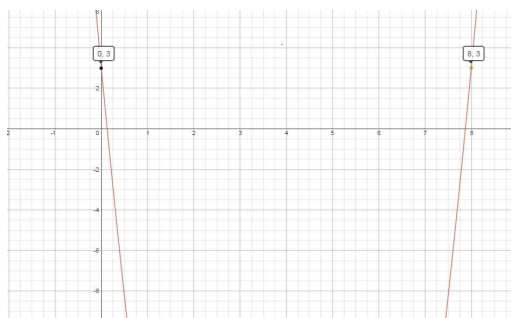
$$x = \frac{0+8}{2} \\ = 4$$

Vertex?

$$f(4) = 3(4)^2 - 24(4) + 3 \\ = 48 - 96 + 3 \\ = -45$$

$$\therefore \text{Vertex } (4, -45)$$

Graphically this is what is going on:



$$f(x) = 3x^2 - 24x + 3 \\ = 3x(x - 8) + 3$$

$$x = 0 \quad x = 8$$

$$(0, 3) \quad (8, 3)$$

**Partial Factored Form:**      $f(x) = ax(x - s) + t$

Process:

- From standard form, factor  $ax$  from the first two terms.
- Set  $x = 0$ , then  $y = t$ .  $(0, t)$  is the  $y$ -intercept.
- Set  $x = s$ , then  $y = t$ .  $(s, t)$  is a symmetrical point to the  $y$ -intercept.
- Determine the axis of symmetry.
- Determine the  $y$ -coordinate of the vertex.



The symmetrical points are NOT the Zeros!

Ex. 1 Use partial factoring to determine the vertex.

a)  $f(x) = 2x^2 + 10x + 1$

$$= 2x(x+5) + 1$$

Pt (0, 1)  
Pt (-5, 1)

Axis  
 $x = \frac{0+(-5)}{2}$   
 $= -\frac{5}{2}$

$\therefore$  Vertex  $(-\frac{5}{2}, -\frac{23}{2})$

$$\begin{aligned} \rightarrow f(-\frac{5}{2}) &= 2(-\frac{5}{2})(-\frac{5}{2}+5) + 1 \\ &= -5(-\frac{5}{2} + \frac{10}{2}) + 1 \\ &= -5(\frac{5}{2}) + \frac{2}{2} \\ &= -\frac{25}{2} + \frac{2}{2} \\ &= -\frac{23}{2} \end{aligned}$$

b)  $f(x) = -2x^2 + 8x - 13$

$$= -2x(x-4) - 13$$

$x=0$        $x=4$

Axis  
 $x = \frac{0+4}{2}$   
 $= 2$

$$\begin{aligned} \rightarrow f(2) &= -2(2)^2 + 8(2) - 13 \\ &= -8 + 16 - 13 \\ &= -5 \\ \therefore (2, -5) \end{aligned}$$

c)  $f(x) = -x^2 - 5x - 3$

$$= -x(x+5) - 3$$

$x=0$        $x=-5$

Axis  
 $x = \frac{0+(-5)}{2}$   
 $= -\frac{5}{2}$

$$\begin{aligned} \rightarrow f(-\frac{5}{2}) &= -(-\frac{5}{2})^2 - 5(-\frac{5}{2}) - 3 \\ &= -(\frac{25}{4}) + \frac{25}{2} - 3 \end{aligned}$$

$$= -\frac{25}{4} + \frac{50}{4} - \frac{12}{4}$$

$$= \frac{13}{4}$$

$\therefore$  Vertex  $(-\frac{5}{2}, \frac{13}{4})$

d)  $f(x) = 5x^2 - 2x + 1$

## **Homework**

### **Handout 1.4B**

**Part A: #1acdf, 2def**

**Part B: #1,2**

**Find vertex by either  
completing the square or  
partial factoring  
- your choice**