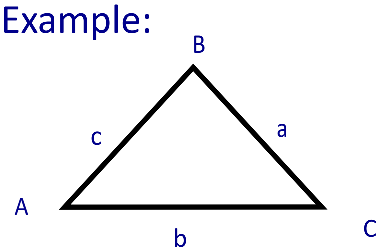
1.2 Similar Triangles

Labelling Non-Right Triangles

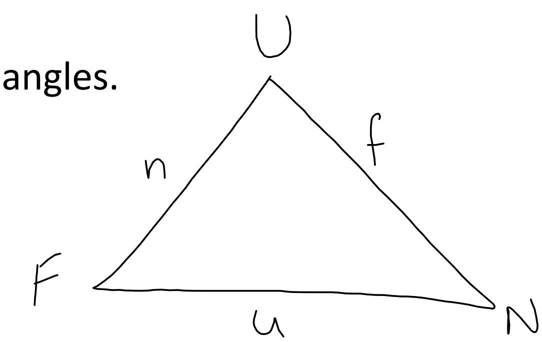
- Angles are denoted by capital letters
- Sides are denoted by lowercase letters

Example:

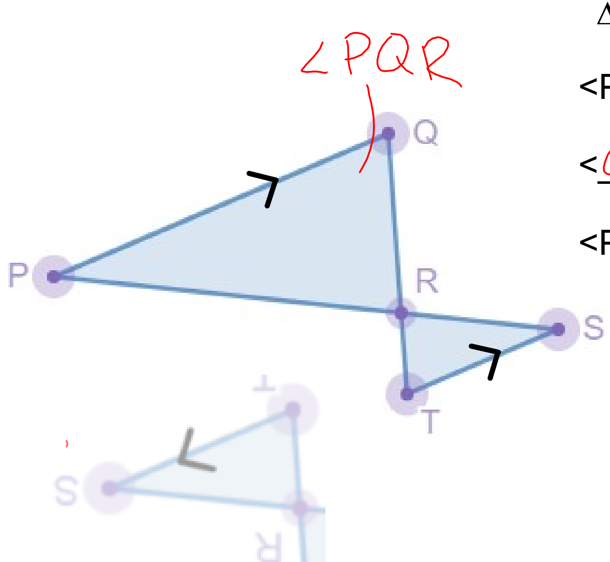


- side 'a' is opposite angle A
- the smallest angle is opposite the smallest side
- the largest angle is opposite the longest side
- the sum of the 2 smaller sides must be greater than the 3rd side

1. Draw triangle FUN. Label the sides and angles.



2.



$\Delta RST \sim \Delta RPQ$

<PQR = < STR

<QPR = <TSR

<PRQ = < SRT

$\frac{PQ}{ST} = \frac{?}{RS}$ RP

$\frac{TR}{QR} = \frac{TS}{?}$ QP

$\frac{RQ}{?} = \frac{PR}{SR}$

RT

When 2 triangles are similar, we say that ΔABC ~ΔDEF.
The order of the letters means that:

< A = ∠D

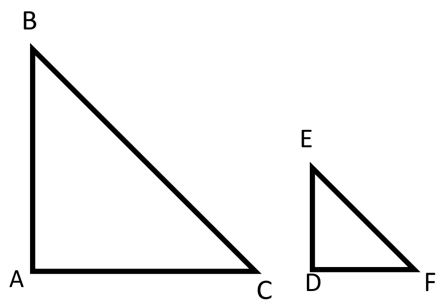
< B = ∠E and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

< C = ∠F

Ex. 1 Complete the statements about the pair of similar triangles.

a)

$\Delta ABC \sim \Delta DEF$



<A = < D

<B = < E

<C = < F

These are CORRESPONDING ANGLES

big Δ → $\frac{AB}{?\ DE} = \frac{BC}{EF}$ ← big Δ
small Δ → ← small Δ

big Δ → $\frac{BC\ ?}{EF} = \frac{AC}{DF}$ ← big Δ
small Δ → ← small Δ

big Δ → $\frac{AB}{DE} = \frac{AC}{?}$ DF ← big Δ
small Δ → ← small Δ

Two triangles are similar if:

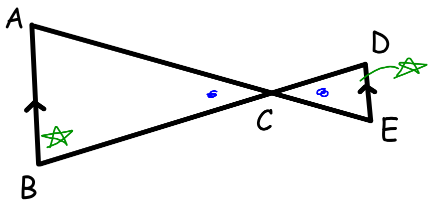
- the corresponding angles are equal.

OR

- the lengths of the corresponding sides are proportional.

Ex. 2 Determine if the following pairs of triangles are similar.

a)

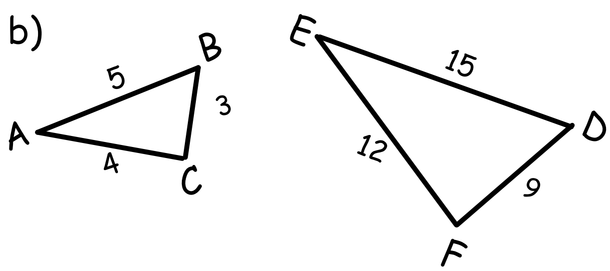


$\angle DCE = \angle BCA$ (Opposite angles O.A.T.)

$\angle B = \angle D$ (Z-pattern)

$\therefore \triangle ABC \sim \triangle EDC$

b)



Are sides proportional?

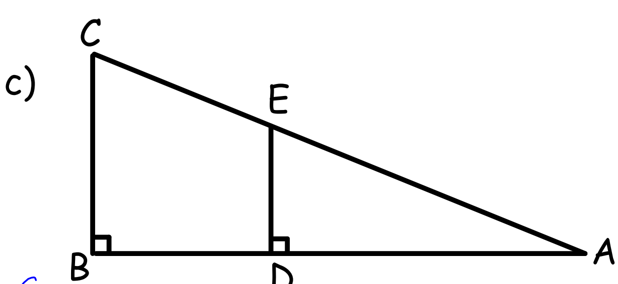
$\frac{ED}{AB} = \frac{15}{5} = 3$ $\frac{DF}{BC} = \frac{9}{3} = 3$

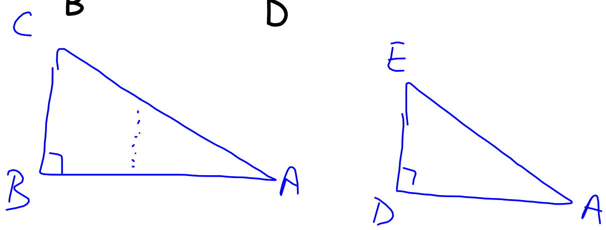
$\frac{EF}{AC} = \frac{12}{4} = 3$

$\therefore$ All are proportional!

$\therefore \triangle ABC \sim \triangle EDF$

c)





Can we confirm angles are the same?

$\angle B = \angle D$

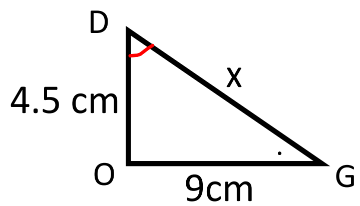
$\angle A = \angle A$

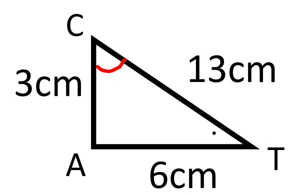
$\angle C = \angle E$ (F-pattern)

$\therefore \triangle CBA \sim \triangle EDA$

Ex. 3 Prove that the following triangles are similar. Determine the unknowns.

a)





Prove?

- Angles are same

OR

- Proportional sides

Given $\angle D = \angle C$

$\angle G = \angle T$

$\therefore \triangle DOG \sim \triangle CAT$

Solve

$$\frac{DO}{CA} = \frac{DG}{CT}$$

$$\frac{4.5}{3} = \frac{x}{13}$$

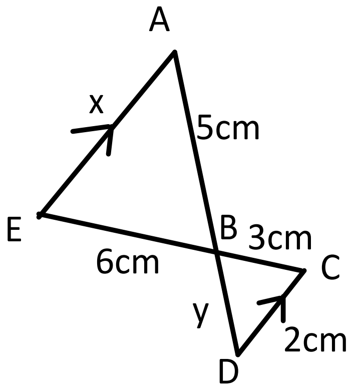
$$13\left(\frac{4.5}{3}\right) = x$$

$$x = 19.5$$

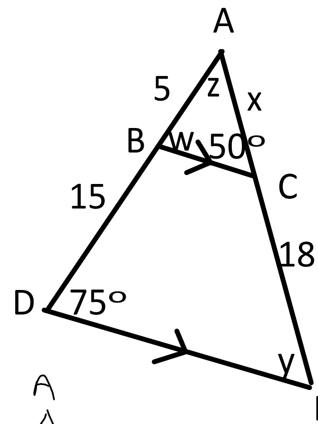
$\therefore x = 19.5\text{cm}$

Ex. 4 Prove that the following triangles are similar. Determine the unknowns.

a)



b)



Prove

$\angle EBA = \angle CBD$ (O.A.T.)

$\angle A = \angle D$ (Z-pattern)

$\therefore \triangle ABE \sim \triangle DBC$

Solve

$$\frac{EA}{CD} = \frac{EB}{CB}$$

$$\frac{x}{2} = \frac{6}{3}$$

$$x = 2\left(\frac{6}{3}\right)$$

$$= 4$$

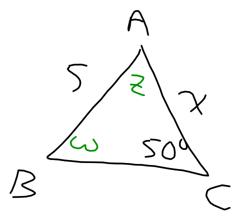
$$\frac{DB}{AB} = \frac{CB}{EB}$$

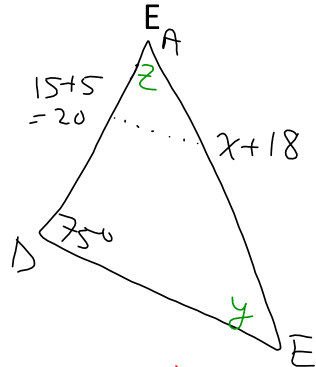
$$\frac{y}{5} = \frac{3}{6}$$

$$y = 5\left(\frac{3}{6}\right)$$

$$= \frac{15 \div 3}{6 \div 3}$$

$$= \frac{5}{2}$$





Prove?

$\angle A = \angle A$

$\angle B = \angle D$ (F-pattern)

Solve

$\boxed{w = 75°}$

$z = 180 - 75 - 50$

$\boxed{z = 55°}$

$y = 180 - 75 - 55°$

$\boxed{y = 50}$

$$\frac{x}{x+18} = \frac{5}{15}$$

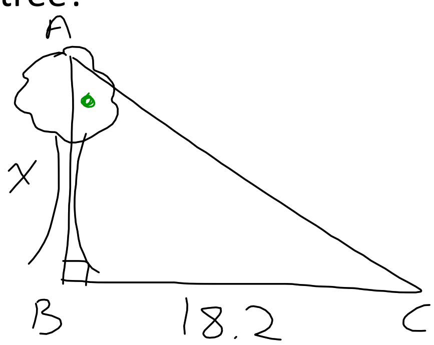
$$15x = 5(x+18)$$

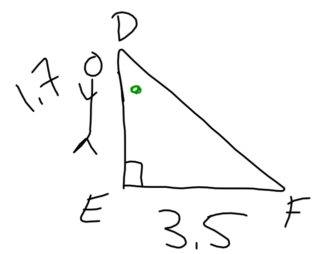
$$15x = 5x + 90$$

$$10x = 90$$

$$\boxed{x = 9}$$

Ex. 5 On a sunny day, Tanner who is 1.7 m tall and standing by a tree, casts a shadow which is 3.5 m long. The nearby tree casts a shadow of 18.2 m long. How tall is the tree?





Prove?

$\angle B = \angle E$

$\angle A = \angle D$ (Sunlight)

Solve

$$\frac{x}{1.7} = \frac{18.2}{3.5}$$

$\vdots$

$x = 8.84$

$\therefore$ The tree is 8.8 m tall.

Homework

Set 1: p. 348 #6e, 9, 11, 14

Set 2: p. 348 #6e, 9, 14, 23ab*, 24