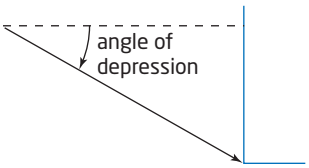
7.5 Solve Problems Involving Right Triangles

The primary trigonometric ratios are applied in many areas of study, including architecture, engineering, astronomy, medicine, and criminal detection. For example, have you ever seen a television show or movie in which forensic evidence is used to recreate the events of an accident or crime? How can scientists use a few tiny clues, such as bloodstains, to piece together what actually happened? How could trigonometry be useful?

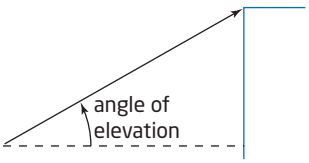
angle of depression

- angle measured below the horizontal
- also called the **angle of declination**



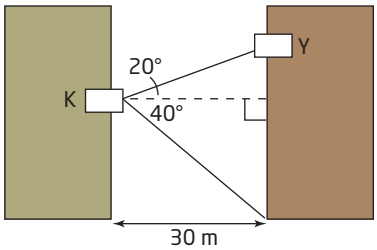
angle of elevation

- angle measured above the horizontal
- also called the **angle of inclination**



Example 1 Angles of Depression and Elevation

Kim and Yuri live in apartment buildings that are 30 m apart, as shown. The **angle of depression** from Kim's balcony to where Yuri's building meets the ground is 40°. The **angle of elevation** from Kim's balcony to Yuri's balcony is 20°.

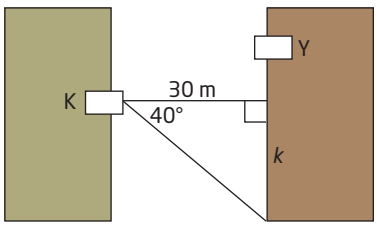


a) How high is Kim's balcony above the ground, to the nearest metre?

b) How high is Yuri's balcony above the ground, to the nearest metre?

Solution

a) Simplify the problem by focusing on the lower right triangle.



$$\tan 40° = \frac{\text{opposite}}{\text{adjacent}}$$

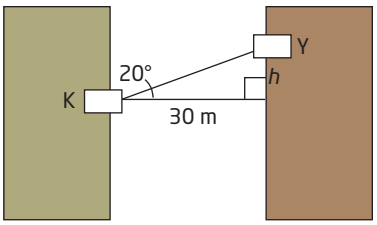
$$\tan 40° = \frac{k}{30}$$

$30 \tan 40° = k$ Multiply both sides by 30.

$25.17 \doteq k$

Kim's balcony is about 25 m above the ground.

b) Focus on the upper right triangle.



$$\tan 20° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 20° = \frac{h}{30}$$

$$30 \tan 20° = h$$ Multiply both sides by 30.

$$10.92 \doteq h$$

Yuri's balcony is about 11 m higher than Kim's. Add this to the height of Kim's balcony to find the height of Yuri's balcony, y.

$y = h + k$

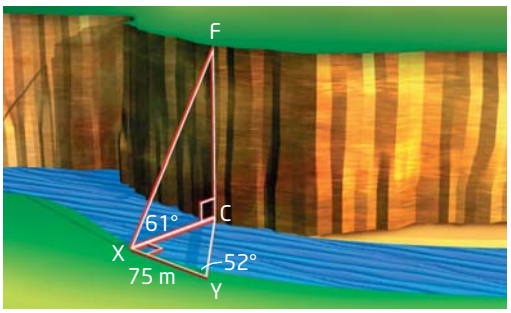
$= 25 + 11$

$= 36$

Yuri's balcony is about 36 m above the ground.

Example 2 Solve a Three-Dimensional Problem

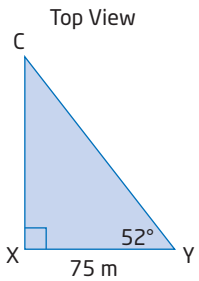
A theodolite is an instrument used by a surveyor to measure horizontal and vertical angles. Measurements are taken in order to find the height of a cliff on the other side of a river, as shown.



Find the height of the cliff, to the nearest metre.

Solution

Use △CYX to find the width of the river, CX. Then, use △CXF to find the height of the cliff, CF. Focus on △CYX first.



Apply the tangent ratio to find CX.

$$\tan \angle Y = \frac{CX}{XY}$$

$$\tan 52° = \frac{CX}{75}$$

$$75 \tan 52° = CX$$ Multiply both sides by 75.

$$95.996 \doteq CX$$

CX is shared by both triangles. Use it to find the height of the cliff.

Literacy Connections

In this situation, using a single letter to identify a side can lead to confusion. For example, side c could refer to the hypotenuse in △CXF or the 75-m side of △CYX. Use endpoints to distinguish line segments, FX and XY in this case, to avoid confusion.

Focus on $\triangle CXF$.

Apply the tangent ratio to find CF.

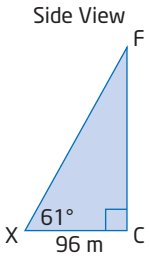
$$\tan \angle X = \frac{CF}{CX}$$

$$\tan 61° = \frac{CF}{96}$$

$$96 \tan 61° = CF$$ Multiply both sides by 96.

$$173.2 \doteq CF$$

The cliff is about 173 m high.



Key Concepts

- Angles of elevation and depression are measured above and below a horizontal line, respectively.
- The primary trigonometric ratios can be applied to solve two-dimensional and three-dimensional problems involving right triangles.
- Some complex problems involve working with more than one right triangle.

Communicate Your Understanding

C1 **a)** What is an angle of elevation?

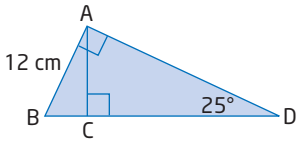
b) How is it measured?

C2 **a)** What is an angle of depression?

b) How is it measured?

C3 **a)** What triangles can you identify in the diagram?

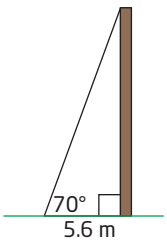
b) Describe the steps you would use to find the side length CD.



Practise

For help with questions 1 and 2, see Example 1.

1. A telephone pole is secured at its top with a guy wire, as shown. The guy wire makes an angle of 70° with the ground and is secured 5.6 m from the bottom of the pole. Find the height of the telephone pole.



2. a) Find the length of the guy wire in question 1.

b) Use a different method to find the length of the guy wire in question 1.

For help with questions 3 to 7, see Example 2.

3. Refer to question 1. A second guy wire is to be added to support the pole. It is to be secured on the ground twice as far from the pole as the first wire, on the same side of the pole and attached to the top of the pole.

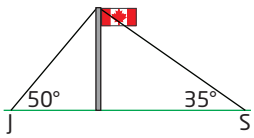
a) Draw a diagram illustrating the telephone pole and both guy wires.

b) Find the length of the second wire and the angle it will make with the ground.

c) Find the angle formed between the two wires at the top of the pole.

4. Refer to question C3. Solve for side length CD. Record your answer to the nearest centimetre.

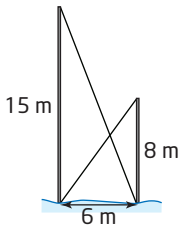
5. Jack and Sangita are facing each other on opposite sides of a 10-m flagpole. From Jack's point of view, the top of the flagpole is at an angle of elevation of 50°. From Sangita's it is 35°.



How far apart are Jack and Sangita?

6. Alexa and Emma are looking up at their house from the backyard. From Alexa's point of view, the top of the house is at an angle of elevation of 40°. From Emma's point of view, directly closer to the house, it is 60°. The house is 15 m high. How far apart are the two girls?

7. At the bottom of a ski lift, there are two vertical poles: one 15 m tall and the other 8 m tall. The ground between the poles is level, and the bases of the poles are 6 m apart. The poles are connected by two straight wires.



a) What angle does each wire make with the ground?

b) What is the length of each wire?

8. Refer to question 7.

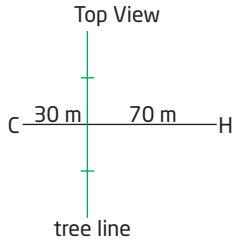
a) Use a different method to answer part b).

b) At what height above the ground do the wires intersect, to the nearest tenth of a metre?

Connect and Apply

Use this information to answer questions 9 and 10.

Cheryl is golfing. She is 100 m from the hole, which is her target. Blocking her direct path is a line of trees, the midpoint of which is 30 m from her current position. She has two choices:

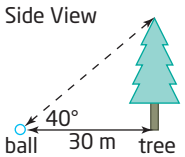


• Option 1: Aim directly for the hole, over the trees.
• Option 2: Go around the trees in two shots.

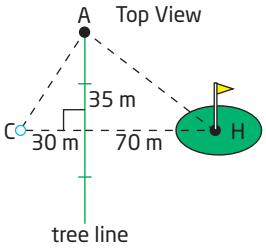
Cheryl's average distances using various clubs are shown in the table.

Club	Distance (m)
Lob wedge	25
Sand wedge	50
Pitching wedge	90
9-iron	100

9. Cheryl considers Option 1, to aim over the trees. The closest tree, which is in line with the hole, is about 30 m away, as shown. She estimates that the angle of inclination from her ball to the top of the tree is about 40°. She judges that the maximum height she can hit from this position is 20 m. Should she take this shot? Explain why or why not.



10. Cheryl considers Option 2, to go around the tree in two shots. In shot one, she will shoot to the end of the line of trees to position A. In shot two, she will aim for the hole.



The tree line is 70 m long, and the line joining Cheryl's ball and the hole, CH, passes through the middle of the tree line, at a right angle.

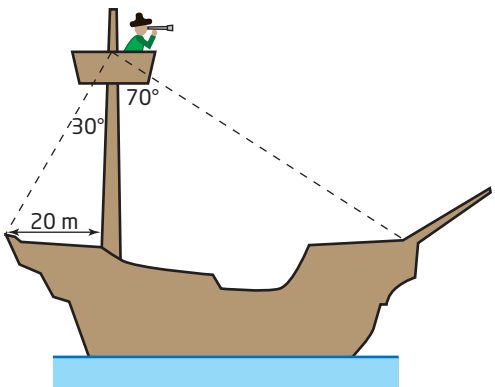
a) At what angle from CH should Cheryl make her first shot, in order to land near A?

b) Which club should she choose for this shot? Explain.

c) Assume that Cheryl succeeds with her first shot and her golf ball lands at A. At what angle from the tree line must she aim for her second shot?

d) Which club should she choose for this shot? Explain.

11. Captain Jack is sitting in the crow's-nest of his ship, as shown.

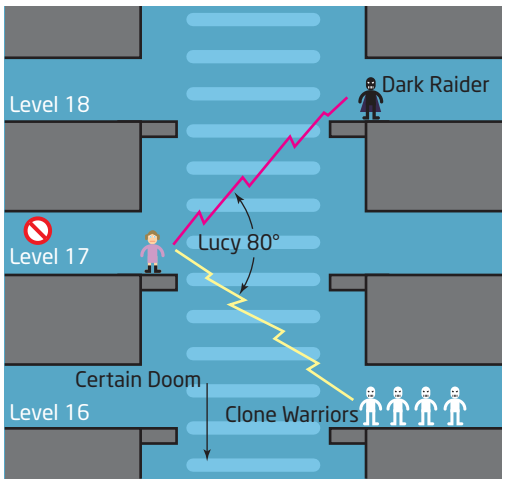


a) How high above the deck is Captain Jack?

b) What is the length of Captain Jack's ship?

c) How long is each wire holding up the crow's-nest?

12. Lucy Starstrider is trapped on Level 17 of a space station. The evil Dark Raider and his Clone Warriors, who are on Levels 18 and 16 across a deep trench, face her. Lucy's retreat path behind her has been blocked. Her only chance for escape is to try to jump directly across the trench. If she does not make it, she will fall to certain doom.



Lucy estimates that the lines of sight to her enemies are 80° apart, as shown. She also recalls, from the technical plans of the space station, that consecutive levels are 10 m apart, vertically. Using the Source of Power, Lucy can leap a horizontal distance of 12 m. Time is quickly running out! Will Lucy Starstrider escape the clutches of the evil ones, or will she perish? Justify your answer.

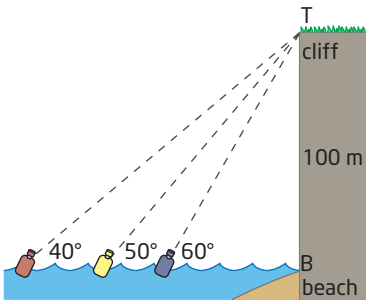
13. A scuba diver swam north at 1.5 m/s, across a current running from east to west at 2.0 m/s. She swam for 3 min and then surfaced.

a) Draw a diagram showing where the dive boat will pick her up relative to where she dove.

b) How far did she travel?

14. The observation deck at Peggy's Cove lighthouse, in Nova Scotia, is about 20 m above sea level. From the observation deck, the angle of depression of a boat on the water is 6°. How far is the boat from the lighthouse, to the nearest metre?

15. Theresa and Branko are competing in a series of outdoor challenges that will eventually lead them to a hidden treasure. Each clue they find helps them find a new clue. Theresa is at the top of a cliff that she knows to be 100 m high, looking down at three anchored floating bottles, as shown.



She reads the clue that she and Branko just found:

> From the top of the cliff, find the bottle whose angle of depression is 50°.

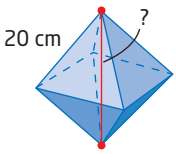
Branko is waiting on the beach below for instructions from Theresa.

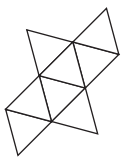
a) What colour bottle should Theresa tell Branko to look for?

b) How far out should she tell him to swim?

16. Refer to Example 1. Yuri's balcony is four floors above Kim's. What floors do they live on? Explain any assumptions you must make.

17. An octahedron is formed by attaching eight congruent equilateral triangles, as shown.

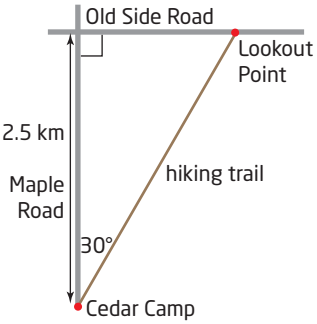




net of octahedron

If the length along one of the edges is 20 cm, find the distance between opposite vertices.

18. Joanne and Sandy are hiking from Cedar Camp to Lookout Point along the hiking trail shown.



Cedar Camp is 2.5 km from Old Side Road along Maple Road, which runs flat. The hiking trail makes an angle of 30° with Maple Road and climbs at an average angle of elevation of 15°.

a) How far apart would Cedar Camp and Lookout Point appear, according to a normal map?

b) What distance do the hikers actually walk? Why are these answers different?

c) What is the difference in elevation between Lookout Point and Cedar Camp?

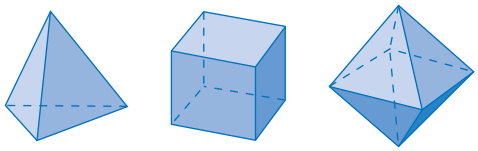
d) What is the average angle of elevation of the section of Old Side Road that is shown?

Making Connections

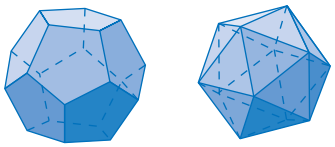
The octahedron is one of five Platonic solids, polyhedra that are formed by attaching congruent regular polygons.

The Platonic solids are named after Plato, a great Greek mathematician and teacher, who lived from 427 to 347 B.C.E.

Plato is famous for founding the world's first known university in Athens, Greece, called The Academy.

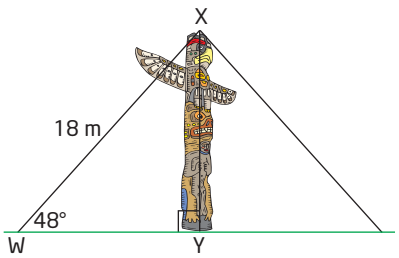


tetrahedron cube octahedron



dodecahedron icosahedron

19. Ropes are used to pull a totem pole upright. Then, the ropes are anchored in the ground to hold the pole until the hole is filled. One of the ropes holding this totem pole is 18 m long and forms an angle of 48° with the ground.



a) Find the height of the totem pole, to the nearest metre.

b) How far is the anchor point from the base of the totem pole, to the nearest metre?

20. Edmonton's CN Tower is a highrise office building. From a point 35 m from the base of the building and level with the base, the angle of elevation of the top is 72.5°. Find the height of Edmonton's CN Tower, to the nearest metre.

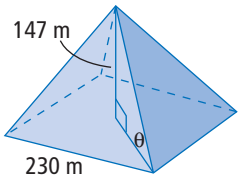
21. A coast guard patrol boat is 14.8 km east of the Brier Island lighthouse. A disabled yacht is 7.5 km south of the lighthouse.

a) How far is the patrol boat from the yacht, to the nearest tenth of a kilometre?

b) At what angle south of due west, to the nearest degree, should the patrol boat travel to reach the yacht?

22. The Capilano Suspension Bridge in North Vancouver is the world's highest footbridge of its kind. The bridge is 140 m long. From the ends of the bridge, the angles of depression of a point on the river under the bridge are 41° and 48°. How high is the bridge above the river, to the nearest metre?

23. The Great Pyramid of Cheops is a square-based pyramid with a height of 147 m and a base length of 230 m. Find the angle, to the nearest degree, that one of the edges of the pyramid makes with the base.



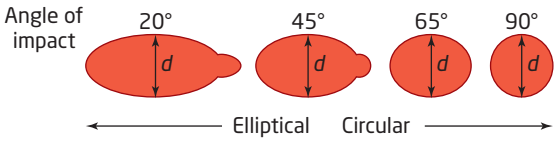
24. Chapter Problem The city you are now in is next to a large river. Take a boat downstream to another city located at the river's end. From there you must fly to a town known as the Gateway to the Yukon. This town, your current location in the race, and Canada's capital form a right triangle. The tangent of the angle at the city you have just sailed to is 1.75. What is your final destination?

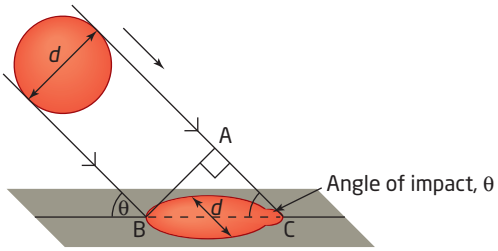
Extend

Use this information to answer questions 25 and 26.

Forensic scientists can recreate an accident or crime by examining bloodstains. A blood droplet starts out in the shape of a sphere. When it falls straight down to the floor, it usually forms a circle with the same diameter as the sphere.

However, when blood hits the floor at an angle, due to the force of a blow, the circle becomes elongated into a shape called an ellipse. The ellipse's width is the same as the sphere's diameter, but because of the force in the direction of motion, its length is greater than the sphere's diameter.

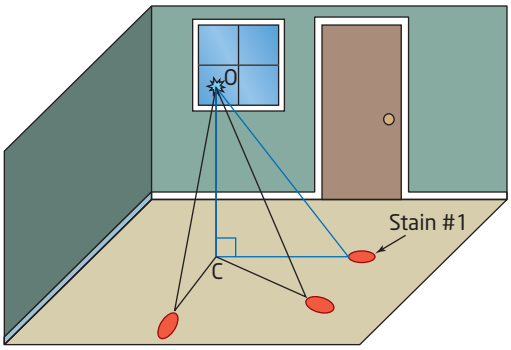




The angle of impact, θ, is the acute angle formed between the path of the blood drop and the floor.

25. Use geometric reasoning to show that the angle of impact can be found using the relationship $\sin\theta = \frac{d}{BC}$.

26. Three bloodstains from a victim are shown. The point of convergence, C, has been found by extrapolating the directions of these stains along the floor. The origin of the blow, O, is some height above C.



Forensic analysis of Stain #1 provides the following data.

```
Length of bloodstain: ..... 4.2 cm
Width of bloodstain: ..... 2.6 cm
Distance from point
of convergence ..... 2.1 m
```

Determine the height at which the blow struck the victim.

27. A sign shows that a hill has a grade of 9%. What angle does the hill make with the horizontal, to the nearest tenth of a degree?

28. An airplane is cruising at an altitude of 10 000 m. It is flying in a straight line away from Chandra, who is standing on the ground. If she sees the angle of elevation of the airplane change from 70° to 33° in 1 min, what is its cruising speed, to the nearest kilometre per hour?

29. A special type of aircraft is designed to fly at the very low height of 20 m. To measure such a small altitude, two spotlights are mounted on the aircraft:

- one on the nose, pointing straight down
- another mounted on the tail of the aircraft, 10 m away

Find the angle at which the second light needs to be set, with respect to the body of the aircraft, so that the beams will meet 20 m below the aircraft.

30. The angle of elevation to a building is 30°. From a point 20 m directly toward the building, the angle of elevation changes to 40°. Find the height of the building. Include a diagram with your solution.

31. **Math Contest** In a family of four children, what is the probability that there are at least two girls if the eldest child is a girl?

A $\frac{15}{16}$ **B** $\frac{7}{16}$ **C** $\frac{3}{4}$ **D** $\frac{7}{8}$ **E** $\frac{3}{8}$

32. In the diagram, $\triangle ABC$ is isosceles, with AB = AC, and $\triangle RST$ is equilateral. Express $\angle x$ in terms of $\angle y$ and $\angle z$.