Lesson 1.5A Working With Radicals

$\sqrt[n]{m}$

index

radicand

refers to some irrational numbers (radicand not a perfect square)

Properties of Radicals:

1) Multiplication

$\sqrt{4 \cdot 25}$
$= \sqrt{100}$
$= 10$

$\sqrt{4} \cdot \sqrt{25}$
$= 2 \cdot 5$
$= 10$

In general: $\sqrt{b} \bullet \sqrt{d}$
$= \sqrt{bd}$

$a\sqrt{b} \bullet c\sqrt{d}$
$= ac\sqrt{bd}$

$2\sqrt{3} \cdot 4\sqrt{5}$
$= 2 \cdot 4\sqrt{3 \cdot 5}$
$= 8\sqrt{15}$

2) Division

$\sqrt{\frac{81}{9}}$
$= \sqrt{9}$
$= 3$

$\frac{\sqrt{81}}{\sqrt{9}}$
$= \frac{9}{3}$
$= 3$

In general: $\frac{\sqrt{b}}{\sqrt{d}}$
$= \sqrt{\frac{b}{d}}$

$\frac{a\sqrt{b}}{c\sqrt{d}}$
$= \frac{a}{c}\sqrt{\frac{b}{d}}$

$\frac{2\sqrt{18}}{4\sqrt{9}} = \frac{2}{4}\sqrt{\frac{18}{9}}$
$= \frac{1}{2}\sqrt{2}$
$= \frac{\sqrt{2}}{2}$

3) Squaring

$(\sqrt{b})^2$
$= b$

$(a\sqrt{b})^2$
$= a^2(\sqrt{b})^2$
$= a^2 b$

In general:

$(a\sqrt{b})^m$
$= a^m(\sqrt{b})^m$

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Radicals can be:

Entire or **Mixed**

$\sqrt{n}$ $a\sqrt{b}$

Perfect Squares & Square Roots

$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$11^2 = 121$	$\sqrt{121} = 11$
$12^2 = 144$	$\sqrt{144} = 12$
$13^2 = 169$	$\sqrt{169} = 13$
$14^2 = 196$	$\sqrt{196} = 14$
$15^2 = 225$	$\sqrt{225} = 15$

Sometimes entire radicals can be changed to mixed radicals by simplifying.

Ex. 1 Change Entire Radicals to Mixed Radicals

a) $\sqrt{27}$
$= \sqrt{3}\sqrt{9}$
$= \sqrt{3} \cdot 3$
$= 3\sqrt{3}$

b) $\sqrt{48}$
$= \sqrt{16 \cdot 3}$
$= \sqrt{16}\sqrt{3}$
$= 4\sqrt{3}$

c) $\sqrt{500}$
$= \sqrt{100 \cdot 5}$
$= \sqrt{100} \cdot \sqrt{5}$
$= 10\sqrt{5}$

d) $\sqrt{180}$
$= \sqrt{36} \cdot \sqrt{5}$
$= 6\sqrt{5}$

A radical is in <u>simplest form</u> if:

1. The radical has no perfect square factors other than 1 in the radicand.
2. There are no fractions under a $\sqrt{\ }$. $\sqrt{\frac{1}{6}}$
3. There are no $\sqrt{\ }$ in the denominator. $\frac{1}{\sqrt{2}}$

Multiplying and Dividing Radicals:

Ex. 2 Simplify.

a) $\sqrt{5}\bullet\sqrt{7}$

$= \sqrt{35}$

b) $3\sqrt{6}\bullet\sqrt{2}$

$= 3\sqrt{12}$

$= 3\sqrt{4}\sqrt{3}$

$= 3\cdot 2\cdot\sqrt{3}$

$= 6\sqrt{3}$

c) $(5\sqrt{6})(2\sqrt{8})$

$= 10\sqrt{48}$

$= 10\sqrt{16}\sqrt{3}$

$= 40\sqrt{3}$

$(3x)(4x^2)$

$= 12x^3$

d) $(2\sqrt{6})(3\sqrt{2})(5\sqrt{6})$

$= 2\cdot 3\cdot 5\sqrt{6\cdot 2\cdot 6}$

$= 30\sqrt{72}$

$= 30\sqrt{36}\sqrt{2}$

$= 180\sqrt{2}$

e) $\sqrt{3}(\sqrt{6}+5)$

$= \sqrt{18} + 5\sqrt{3}$

$= \sqrt{9}\sqrt{2} + 5\sqrt{3}$

$= 3\sqrt{2} + 5\sqrt{3}$

$2(x+3)$

f) $\dfrac{\sqrt{18}}{\sqrt{3}}$

$= \dfrac{\sqrt{6}\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$

$= \sqrt{6}$

$= \sqrt{\dfrac{18}{3}}$

$= \sqrt{6}$

g) $\dfrac{\overset{5}{\cancel{15}}\sqrt{7}}{\underset{1}{\cancel{3}}\sqrt{4}}$

$= \dfrac{5\sqrt{7}}{\sqrt{4}}$

$= \dfrac{5}{2}\sqrt{7}$

h) $\dfrac{5\sqrt{12}}{\sqrt{8}} = \dfrac{5\cancel{\sqrt{4}}\sqrt{3}}{\cancel{\sqrt{4}}\sqrt{2}}$

$= \dfrac{5\sqrt{3}}{\sqrt{2}}$ $\quad \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{5\sqrt{6}}{2}$

Rationalizing the Denominator

Multiply both the numerator and the denominator by the radical in the denominator.

Squaring the radical will eliminate it from the denominator.

i) $\dfrac{\overset{3}{\cancel{12}}\sqrt{27}}{\underset{1}{\cancel{4}}\sqrt{45}}$

$= \dfrac{3\cancel{\sqrt{9}}\sqrt{3}}{\cancel{\sqrt{9}}\sqrt{5}}$

$= \dfrac{3\sqrt{3}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{3\sqrt{15}}{5}$

Adding and Subtracting Radicals:

Algebra: Collect like terms.

Like Terms

Same variables, same exponents

Example: $2x$, $3x$

Counter-example: x^2, $4x$

Radicals: Collect like radicals.

Like Radicals

Same index, same radicand

Example: $\sqrt{3}$, $3\sqrt{3}$, $5\sqrt{3}$

Counter-example: $2\sqrt{3}$, $2\sqrt{5}$

Ex. 3 Are the following radicals like or unlike?

a) $2\sqrt{3}, -3\sqrt{3}, 4\sqrt{3}$

LIKE

b) $\sqrt{4}, \sqrt{2}, \sqrt{3}$

UNLIKE

c) $\sqrt{8}, \sqrt{2}, \sqrt{32}$

$2\sqrt{2}$ $4\sqrt{2}$

LIKE

d) $\sqrt[3]{3}, \sqrt{3}, \sqrt[4]{3}$

UNLIKE

Ex. 4 Add or Subtract.

a) $\sqrt{27} + \sqrt{20} - \sqrt{12} + \sqrt{45}$

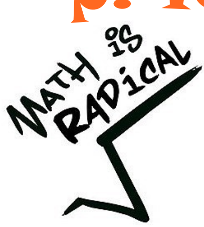
$= 3\sqrt{3} + 2\sqrt{5} - 2\sqrt{3} + 3\sqrt{5}$

$= \sqrt{3} + 5\sqrt{5}$

b) $7\sqrt{2} - 6\sqrt{63} - \sqrt{28} + 5\sqrt{18}$

$= 22\sqrt{2} - 20\sqrt{7}$

Homework
p. 167 # 1-7, 15ab, 16



$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$

$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

You CANNOT split up the radical across a + or – sign.

$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$

$\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$