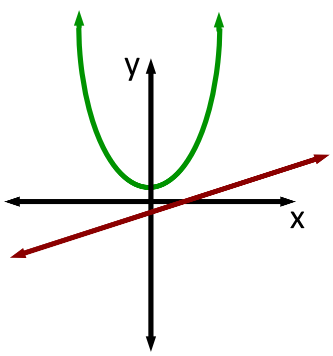
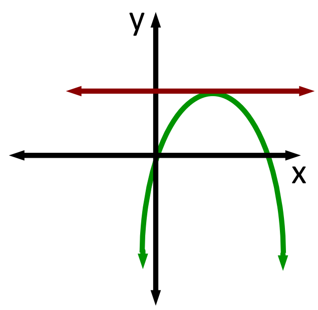
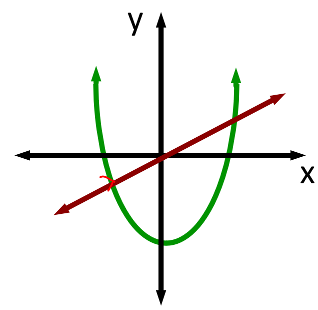
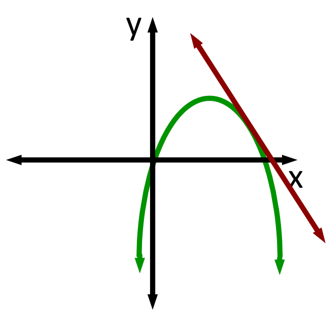
1.8 Solving Linear and Quadratic Systems

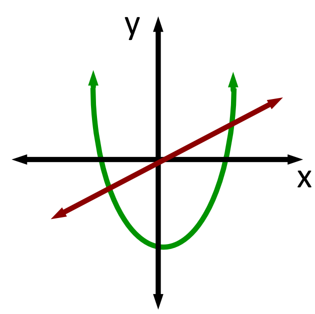
A <u>system</u> of equations consists of <u>two or more</u> equations. If the graphs in the system are linear (degree 1) and quadratic (degree 2), the system could have <u>no solution</u>, <u>one solution</u>, or <u>two solutions</u>.

<u>**No Solution**</u>	<u>**One Solution**</u>	<u>**Two Solutions**</u>
		

<u>Tangent</u> - A line that intersects a curve at <u>one</u> point and has the same slope as the curve at that point.



<u>Secant</u> - A line that intersects a curve at <u>two</u> distinct points.



Process for solving a linear-quadratic system algebraically:

1. Isolate one variable from the linear equation.
2. Sub into the quadratic.
3. Solve for the remaining variable.
4. Sub answer(s) back into the linear equation to find the coordinate(s) of intersection, if they exist.

Ex. 1 Solve the system.

① $y = x^2 - 3$

② $2x + y = -3$

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Step 1

From ②

$y = -2x - 3$

Step 2 - Sub into ①

$-2x - 3 = x^2 - 3$

Step 3 - Solve

$0 = x^2 + 2x$

$= x(x+2)$

$x = 0$ $x = -2$

Step 4 - Sub in to solve for y(s)

② $2x + y = -3$

$x = 0$

$2(0) + y = -3$

$y = -3$

$\therefore (0, -3)$

$x = -2$

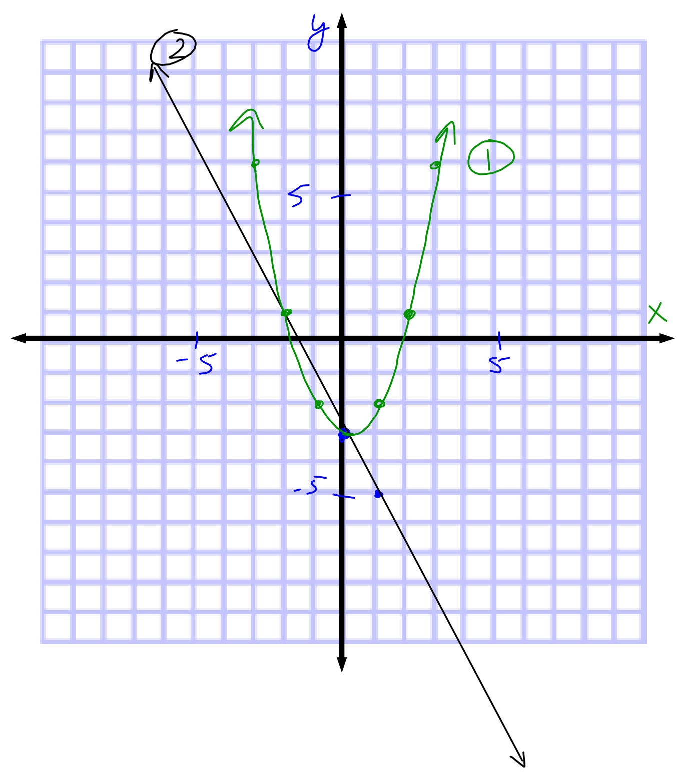
$2(-2) + y = -3$

$y = -3 + 4$

$= 1$

$\therefore (-2, 1)$

Graphically



Ex. 2 Find the coordinates of the point of intersection between the parabola $y-4 = -(x+1)^2$ and the line $y = 3x + 13$.

① $y-4=-(x+1)^2$

② $y=3x+13$

<u>Sub ② into ①</u>

$$3x+13-4=-(x+1)^2$$
$$3x+9=-(x^2+2x+1)$$
$$0=-x^2-2x-1-3x-9$$
$$=-x^2-5x-10$$

QUAD!

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{5\pm\sqrt{(-5)^2-4(-1)(-10)}}{2(-1)}$$

$$=\frac{5\pm\sqrt{25-40}}{-2}$$

Negative!

$\therefore$ NO SOLUTIONS

Ex. 3 If a line with a slope of 4 has one point of intersection with the quadratic function $y=\frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

① $y = 4x + b$

② $y = \frac{1}{2}x^2 + 2x - 8$

Set up to solve the same way, but use descriminant to solve for only 1 solution.

Sub ① into ②

$4x + b = \frac{1}{2}x^2 + 2x - 8$

$0 = \frac{1}{2}x^2 - 2x - 8 - b$

$a = \frac{1}{2}$ $b = -2$ $c = -8 - b$

Use $D = 0$ to solve for b

$D = b^2 - 4ac$

$0 = (-2)^2 - 4(\frac{1}{2})(-8 - b)$

$= 4 - 2(-8 - b)$

$= 4 + 16 + 2b$

$-20 = 2b$

$b = -10$

$\therefore$ y-int is -10

$y = 4x - 10$

Homework

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