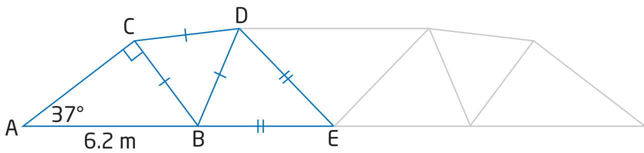
# 8.4 Solve Problems Using Trigonometry

Engineers, scientists, and architects apply a variety of mathematical tools, including trigonometry. When solving problems or creating designs, they must be able to efficiently combine algebraic and geometric reasoning. It is also important for them to be able to communicate their ideas to others effectively.

## Example 1 Bridge Truss

A section of a bridge truss design is shown. Find the total length of the beams required to build the section, to the nearest tenth of a metre.




### Solution

The section of the truss consists of three triangles. Solve for each side length using trigonometry and other mathematical tools.

Start with the triangle with the most given information, △ABC.

Notice that △ABC is a right triangle. First, use the sine ratio to find $a$. Then, use the cosine ratio to find $b$.

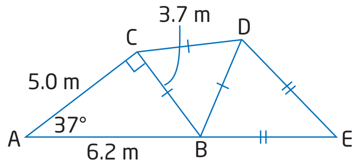
$$\frac{a}{c} = \sin A$$
$$\frac{a}{6.2} = \sin 37°$$
$$a = 6.2(\sin 37°)$$
$$a \doteq 3.7$$

$$\frac{b}{c} = \cos A$$
$$\frac{b}{6.2} = \cos 37°$$
$$b = 6.2(\cos 37°)$$
$$b \doteq 5.0$$

I can use the Pythagorean theorem to check.

$$c^2 = 3.7^2 + 5.0^2$$
$$c^2 = 38.44$$
$$c \doteq 6.2$$

Label these lengths on the diagram.




**Focus on △BCD:**

Notice that △BCD is an equilateral triangle. All sides have equal length. Therefore,
BC = CD = DB = 3.7 m

**Focus on △BDE:**

Notice that △BDE is an isosceles triangle. You can apply the sine law, but you need to find ∠EBD first. The three angles at point B are supplementary.

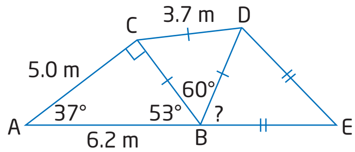
In △ABC, ∠CBA and ∠BAC are complementary because the third angle is 90°.

$$\begin{aligned}\angle CBA &= 90° - 37° \\ &= 53°\end{aligned}$$

Since △BCD is an equilateral triangle, the three interior angles are equal. Therefore,
∠CBD = 60°

Use the two known angles at B to find ∠DBE.

$$\begin{aligned}53° + 60° + \angle DBE &= 180° \\ \angle DBE &= 180° - 53° - 60° \\ \angle DBE &= 67°\end{aligned}$$




△BDE is isosceles with BE = DE. So, ∠EDB = ∠DBE = 67°.

Use this information to find ∠E.

$$\begin{aligned}\angle E &= 180° - 2(67°) \\ &= 46°\end{aligned}$$

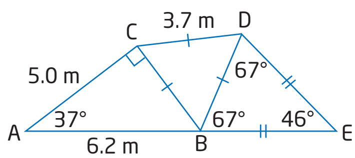
Now use the sine law in △BDE to find the lengths of beams DE and BE.

$$\frac{b}{\sin B} = \frac{e}{\sin E}$$

$$\frac{b}{\sin 67°} = \frac{3.7}{\sin 46°}$$

$$b = \sin 67°\left(\frac{3.7}{\sin 46°}\right)$$

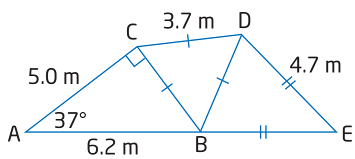
$$b \doteq 4.7$$




To find the total length of the beams required to build this section of the truss, add all the lengths.

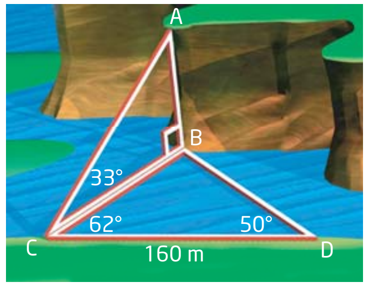
$$\begin{aligned}\text{Total length} &= 5.0 + 6.2 + 3(3.7) + 2(4.7) \\ &= 31.7\end{aligned}$$

The total length of the beams required is 31.7 m.



## Example 2 Height of a Cliff

Find the height of the cliff shown, to the nearest metre.




### Solution

There is not enough given information in △ABC to solve for the height directly. Use the given information in △BCD to solve for the width of the river, BC. Then, use this to find the height of the cliff.

**Focus on △BCD:**

Find the measure of ∠CBD.

$$\begin{aligned}\angle CBD &= 180° - 62° - 50° \\ &= 68°\end{aligned}$$

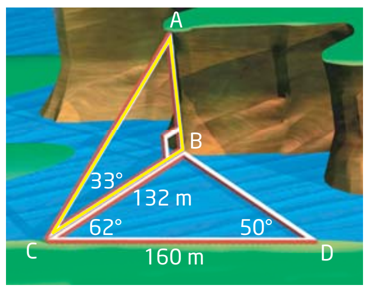
Now, use the sine law to find the width of the river, BC.

$$\begin{aligned}\frac{BC}{\sin BDC} &= \frac{CD}{\sin CBD} \\ \frac{BC}{\sin 50°} &= \frac{160}{\sin 68°} \\ BC &= \frac{160}{\sin 68°}(\sin 50°) \\ BC &\doteq 132\end{aligned}$$

The width of the river is about 132 m. Use this to find the height of the cliff, AB.

**Focus on △ABC:**

When working with right triangles, the sine law and cosine law still apply. However, it is easier to apply the primary trigonometric ratios in right triangles.




$$\begin{aligned}\tan ACB &= \frac{AB}{BC} \\ \tan 33° &= \frac{AB}{132} \\ 132(\tan 33°) &= AB \\ AB &\doteq 86\end{aligned}$$

The height of the cliff is approximately 86 m.

**Literacy Connections**

In complicated diagrams involving more than one triangle, using one letter to identify side lengths can be confusing. In this example, side *b* could mean side AC or side CD. In such cases, use the two endpoints of a line segment to identify it. Similarly, use three letters to identify angles, as needed.

## Key Concepts

- When solving problems involving right triangles, you can apply the primary trigonometric ratios.
- When solving problems involving acute triangles, you can apply the sine law or the cosine law:
  - Use the sine law if you are given an angle and the opposite side, plus one other side or angle.
  - Use the cosine law if you are given two sides and the contained angle, or three sides.
- A number of problems involving trigonometry require multiple steps. Look for techniques and make connections to other branches of mathematics, such as geometry, in order to solve problems efficiently.

## Communicate Your Understanding

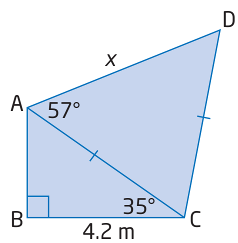
**C1** **a)** Do the sine law and the cosine law hold true in right triangles? Explain.

**b)** What other techniques can you use to solve right triangles?

**C2** Explain how you can decide whether to apply the sine law or the cosine law in an acute triangle.

**C3** **a)** Describe the steps you would take to find the length of $x$ in the diagram shown.

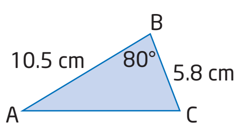
**b)** Describe a different set of steps that will also work.



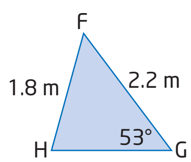

## Practise

1. Determine whether the primary trigonometric ratios, the sine law, or the cosine law should be used first to solve each triangle.

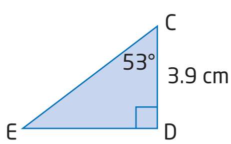
**a)**



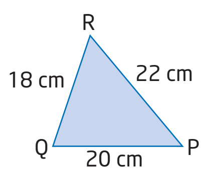

**b)**




**c)**




**d)**




*For help with questions 2 and 3, see Examples 1 and 2.*

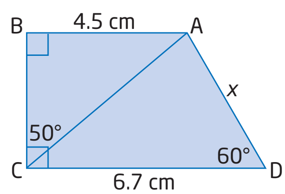
2. Refer to question C3.

   **a)** Use your method from part a) to find $x$, to the nearest tenth of a centimetre.

   **b)** Find $x$ using another method. Compare your answers. Are they equal?

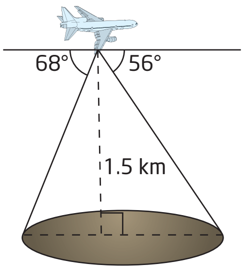
3. **a)** Find $x$, to the nearest tenth of a centimetre.

   **b)** Find $x$ using a different method.



## Connect and Apply

**4.** While flying at an altitude of 1.5 km, a plane measures angles of depression to opposite ends of a large crater, as shown. Find the width of the crater, to the nearest tenth of a kilometre.




**5.** Earth is 149 600 000 km from the Sun. This distance is equal to 1 A.U. (astronomical unit). Mars is 1.5 A.U. from the Sun. One evening, Mars is seen from Earth to make an angle of 68° with the Sun.

**a)** Draw a diagram and label the given information.

**b)** How far apart are Earth and Mars at this point, in kilometres?

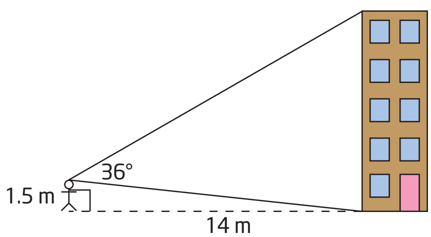
**c)** Do you think the distance between Earth and Mars is always the same? Explain why or why not.

**6.** Lena is in a bicycle road race. In the first leg, she rides 12 km from Riverside to Danton. Then, she turns and rides 17 km to Humberville, making a 74° angle from the first leg. The final turn leads back to Riverside.

**a)** What is the total length of the race, to the nearest kilometre?

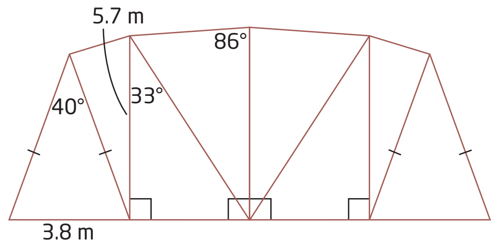
**b)** At what angles are the three towns situated with respect to each other? Round to the nearest degree.

**7.** Trevor, who is 1.5 m tall, is standing at a distance of 14 m from a building. From his point of view, the bottom and top of the building are separated by 36°, as shown. How tall is the building, to the nearest tenth of a metre?



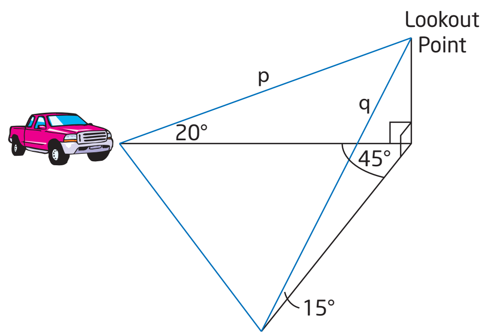

**8.** Rocco and Biff are two koala bears frolicking in a meadow. Suddenly, a tasty clump of eucalyptus falls to the ground, catching their attention. Biff glances at Rocco, who appears to be 15 m away, then over to the eucalyptus, which appears to be 18 m away. From Biff's point of view, Rocco and the eucalyptus are separated by an angle of 45°. Rocco's top running speed is 1.0 m/s, but Biff can run one and a half times as fast. Can Biff beat Rocco to the eucalyptus? State any assumptions you make.

**9.** Find the total length of materials required to build the bridge truss shown, to the nearest tenth of a metre.

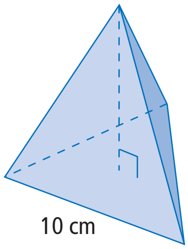



Describe the steps in your solution and state any assumptions you make.

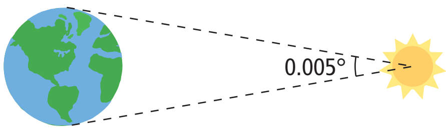
**10.** Lookout Point is accessible from two trails, both of which start from the same altitude and climb upward. Path $p$ travels east to the point and climbs at an average angle of elevation of 20°. Path $q$ travels northeast to the point at an average angle of elevation of 15°. Path $p$ is 2.0 km long. Jack and Debbie parked at the base of path $p$. They hiked a round trip up path $p$ to Lookout Point, then down path $q$, and then finally straight from the base of path $q$ back to their truck. How far did they hike, to the nearest tenth of a kilometre? State any assumptions you make.



**11.** A tetrahedron has edges that are 10 cm in length. Find the height of this tetrahedron, to the nearest tenth of a centimetre.



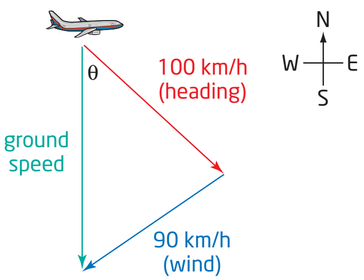

**12.** Doctors Jones and Hwang are astronomers observing the sun from opposite ends of Earth. The radius of Earth is 6400 km.




**a)** Use this information to verify the distance from Earth to the Sun, which was given in question 5. State any assumptions you make.

**b)** At approximately what times of day were these observations made by each astronomer? Explain your answer.

**13. Chapter Problem** Pilots must take wind into account when flying, or the wind will blow them off course and they will not reach the desired destination. Your aircraft cruises at a speed of 100 km/h. There is a strong wind blowing from N60°E at a speed of 90 km/h. You need to fly south to home base.

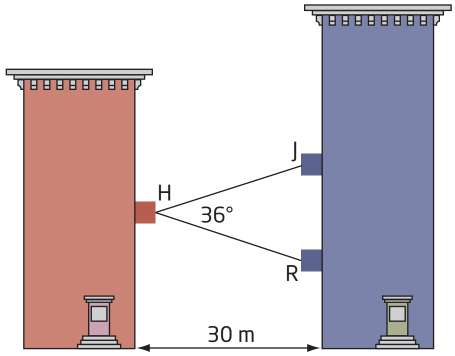



**a)** Find the direction, θ, you must aim the plane, to the nearest degree.

**b)** What will your speed be, over the ground? Round to the nearest unit.

## Extend

**14.** Helen, Javier, and Raquel live in two identical apartment buildings, located 30 m apart. Javier lives two floors higher than Helen. Raquel lives four floors lower than Helen. There is a 36° angle of separation when Helen looks from her balcony to those of her two friends.




**a)** How far apart, vertically, do Javier and Raquel live? Round to the nearest tenth of a metre.

**b)** Explain how you solved this problem and discuss any assumptions you made.

**15.** A box is in the shape of a square-based prism. The height of the box is twice the width of the base.

**a)** Show that the longest thin rod that can be encased in the box has length $\sqrt{6}w$, where $w$ is the width of the base.

**b)** Find the angles that such a rod would make with each edge of the box.

**16.** A ship travels 100 km at a bearing of N60°E and then turns and travels 80 km at a bearing of S20°E before reaching its destination. Suppose the ship travelled directly from its starting point to its destination, following a direct route. What distance and at what bearing would the ship travel? Round to the nearest unit.

**17.** Who uses trigonometry in their careers? Do some research to find out what types of careers require the use of trigonometry and why. Write a brief report of your findings.