

Example 1 Translate Words Into Algebra

- a) Write the following phrase as a mathematical expression:
the value five increased by a number
- b) Write the following sentence as a mathematical equation.
Half of a value, decreased by seven, is one.
- c) Translate the following sentence into an equation, using two variables. Mario's daily earnings are \$80 plus 12% commission on his sales.

Solution

- a) Consider the parts of the phrase.
 - “the value five” means the number 5
 - “increased by” means add or the symbol +
 - “a number” means an unknown number, so choose a variable such as n to represent the number

The phrase can be represented by the mathematical expression $5 + n$.

- b) “Half” means $\frac{1}{2}$
 - “of” means multiply
 - “a value” means a variable such as x
 - “decreased by” means subtract or $-$
 - “seven” is 7
 - “is” means equals or $=$
 - “one” is 1

The sentence can be represented by the equation $\frac{1}{2}x - 7 = 1$.

- c) Consider the parts of the sentence.
 - “Mario's daily earnings” is an unknown and can be represented by E
 - “are” means equals or $=$
 - “\$80” means 80
 - “plus” means +
 - “12% commission on his sales” can be represented by $0.12 \times S$

The sentence translates into the equation $E = 80 + 0.12S$.

Sometimes, several sentences need to be translated into algebra. This often happens with word problems.

Did You Know?

The airplane in Example 2 is a Diamond Katana DA40. These planes are built at the Diamond Aircraft plant in London, Ontario.

Literacy Connections

It is a good idea to read a word problem three times.

Read it the first time to get the general idea.

Read it a second time for understanding. Express the problem in your own words.

Read it a third time to plan how to solve the problem.

Understand the Problem

Choose a Strategy

Carry Out the Strategy

linear system

- two or more linear equations that are considered at the same time

Example 2 Translate Words Into Algebra to Solve a Problem

Ian owns a small airplane. He pays \$50/h for flying time and \$300/month for hangar fees at the local airport. If Ian rented the same type of airplane at the local flying club, it would cost him \$100/h. How many hours will Ian have to fly each month so that the cost of renting will be the same as the cost of flying his own plane?



Solution

Read the paragraph carefully.

What things are unknown?

- the number of flying hours
- the total cost

I'll choose variables for the two unknowns. I will translate the given sentences into two equations. Then, I can graph the two equations and find where they intersect.

Let C represent the total cost, in dollars.
Let t represent the time, in hours, flown.

The first sentence is information that is interesting, but cannot be translated into an equation.

The second sentence can be translated into an equation. Ian pays \$50/h for flying time and \$300/month for hangar fees at the local airport.
 $C = 50t + 300$

The third sentence can also be translated into an equation. If Ian rented the same type of airplane at the local flying club, it would cost him \$100/h.
 $C = 100t$

The two equations form a **linear system**. This is a pair of linear relations, or equations, considered at the same time. To solve the linear system is to find the point of intersection of the two lines, or the point that satisfies both equations.

Graph the two lines on the same grid.

Both equations are in the form $y = mx + b$. You can use the y -intercept as a starting point and then use the slope to find another point on the graph.

The lines on the graph cross at one point, $(6, 600)$. The **point of intersection** is $(6, 600)$.

Check that the solution is correct.

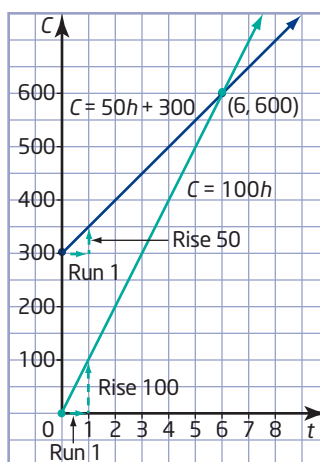
If Ian uses his own airplane, the cost is $6 \times \$50 + \300 . This is \$600.

If he rents the airplane, the cost is $6 \times \$100$. This is \$600.

So, the solution $t = 6$ and $C = 600$ checks.

Write a conclusion to answer the problem.

If Ian flies 6 h per month, the cost will be the same, \$600, for both airplanes.



point of intersection

- a point where two lines cross
- a point that is common to both lines

Reflect

Linear equations are not always set up in the form $y = mx + b$. Sometimes it is easy to rearrange the equation. Other times, you may wish to graph using intercepts.

Example 3 Find the Point of Intersection

The equations for two lines are $x - y = -1$ and $2x - y = 2$. What are the coordinates of the point of intersection?

Solution

Method 1: Graph Using Slope and y -Intercept

Step 1: Rearrange the equations in the form $y = mx + b$.

Equation ①:

$$\begin{aligned} x - y &= -1 \\ x - y + y + 1 &= -1 + y + 1 \\ x + 1 &= y \\ y &= x + 1 \end{aligned}$$

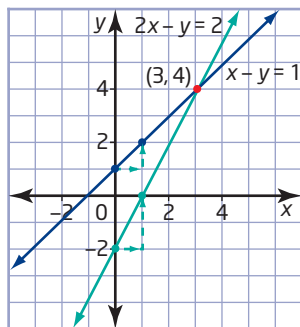
Equation ① becomes $y = x + 1$. Its slope is 1 and its y -intercept is 1.

Equation ②:

$$\begin{aligned} 2x - y &= 2 \\ 2x - y + y - 2 &= 2 + y - 2 \\ 2x - 2 &= y \\ y &= 2x - 2 \end{aligned}$$

Equation ② becomes $y = 2x - 2$. Its slope is 2 and its y -intercept is -2 .

Step 2: Graph and label the two lines.



Step 3: To check that the point (3, 4) lies on both lines, substitute $x = 3$ and $y = 4$ into both original equations.

In $x - y = -1$:

$$\begin{aligned} \text{L.S.} &= x - y & \text{R.S.} &= -1 \\ &= 3 - 4 \\ &= -1 \end{aligned}$$

$$\text{L.S.} = \text{R.S.}$$

So, (3, 4) is a point on the line $x - y = -1$.

In $2x - y = 2$:

$$\begin{aligned} \text{L.S.} &= 2x - y & \text{R.S.} &= 2 \\ &= 2(3) - 4 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

$$\text{L.S.} = \text{R.S.}$$

So, (3, 4) is a point on the line $2x - y = 2$.

If I don't get the same result when I substitute into both equations, I've made a mistake somewhere!

The solution checks in both equations. The point (3, 4) lies on both lines.

Step 4: Write a conclusion.

The coordinates of the point of intersection are (3, 4).

Method 2: Graph Using Intercepts

Step 1: Find the intercepts for each line.

Equation ①: $x - y = -1$

At the x-intercept, $y = 0$.

$$\begin{aligned} x - 0 &= -1 \\ x &= -1 \end{aligned}$$

Graph the point (-1, 0).

At the y-intercept, $x = 0$.

$$\begin{aligned} 0 - y &= -1 \\ -y &= -1 \\ y &= 1 \end{aligned}$$

Graph the point (0, 1).

Equation ②: $2x - y = 2$

At the x-intercept, $y = 0$.

$$\begin{aligned} 2x - 0 &= 2 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

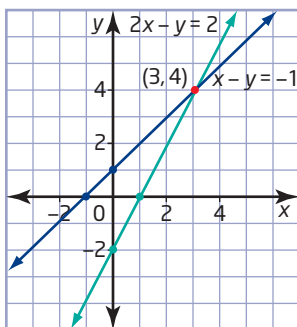
Graph the point (1, 0).

At the y-intercept, $x = 0$.

$$\begin{aligned} 2(0) - y &= 2 \\ -y &= 2 \\ y &= -2 \end{aligned}$$

Graph the point (0, -2).

Step 2: Draw and label the line for each equation.



Step 3: Check by substituting $x = 3$ and $y = 4$ into both original equations.

See Method 1.

Step 4: Write a conclusion.

The coordinates of the point of intersection are $(3, 4)$.

Example 4 Solve an Internet Problem

Brian and Catherine want to get Internet access for their home. There are two companies in the area. IT Plus charges a flat rate of \$25/month for unlimited use. Techies Inc. charges \$10/month plus \$1/h for use. If Brian and Catherine expect to use the Internet for approximately 18 h/month, which plan is the better option for them?

Solution

Represent each situation with an equation. Then, graph to see where the two lines intersect to find when the cost is the same.

Let t represent the number of hours of Internet use.

Let C represent the total cost for the month.

IT Plus:

$$C = 25$$

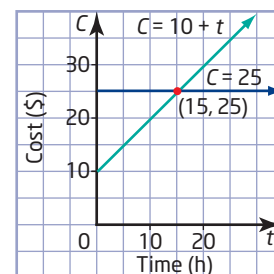
This is a flat rate, which means it costs \$25 and no more.

Techies Inc.:

$$C = 10 + 1t$$

The cost is \$10 plus \$1 for every hour of Internet use.

The two plans cost the same for 15 h of Internet use. The cost is \$25. For more than 15 h, the cost for Techies Inc. Internet service is more than \$25. If Brian and Catherine expect to use the Internet for 18 h/month, they should choose IT Plus.



Example 5 Use Technology to Find the Point of Intersection

Find the point of intersection of the lines $y = x - 12$ and $y = -3x + 20$ by graphing using technology.

Solution

Method 1: Use a Graphing Calculator

- First, make sure that all STAT PLOTS are turned off.
Press 2nd Y= for [STAT PLOT]. Select **4:PlotsOff**.

- Press WINDOW . Use window settings of -20 to 20 for both x and y .

```

WINDOW
Xmin=-20
Xmax=20
Xscl=1
Ymin=-20
Ymax=20
Yscl=1
Xres=1
    
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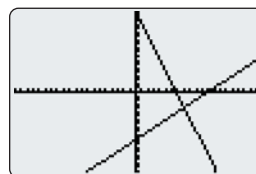
- Enter the two equations as Y1 and Y2 using the Y= editor.

Note: use the - key when entering the first equation, but the (-) key at the beginning of the second equation.

```

Plot1 Plot2 Plot3
Y1=X-12
Y2=-3X+20
Y3=
Y4=
Y5=
Y6=
Y7=
    
```

- Press GRAPH .



- Find the point of intersection using the Intersect function.

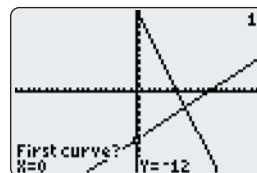
Press 2nd TRACE for the **Calc** menu. Select **5:intersect**.

```

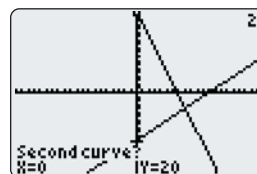
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
    
```

Respond to the questions in the lower left corner.

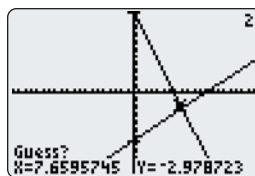
- **First curve?** The cursor will be flashing and positioned on one of the lines. The calculator is asking you if this is the first of the lines for which you want to find the point of intersection. If this is the one you want, press ENTER .



- **Second curve?** The cursor will be flashing and positioned on the second line. The calculator is checking to see if this is the second line in the pair. If this is the line you want, press ENTER .

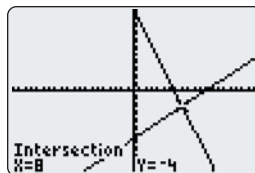


- **Guess?** Here, the calculator is giving you a chance to name a point that you think is the point of intersection. If you do not wish to try your own guess, then press **ENTER** and the calculator will find the point for you.



The point of intersection is $(8, -4)$.

Another way to see the point of intersection is to view the table.



First, press **2nd** **WINDOW** for [TBLSET]. Check that both **Indpnt** and **Depend** have **Auto** selected.

Press **2nd** **GRAPH** for [TABLE].

Cursor down to $x = 8$.

Observe that the values of Y_1 and Y_2 are both -4 at $x = 8$. At other values of x , Y_1 and Y_2 have different values.

X	Y ₁	Y ₂
5	-7	5
6	-6	2
7	-5	-1
8	-4	-4
9	-3	-7
10	-2	-10
11	-1	-13
X=8		

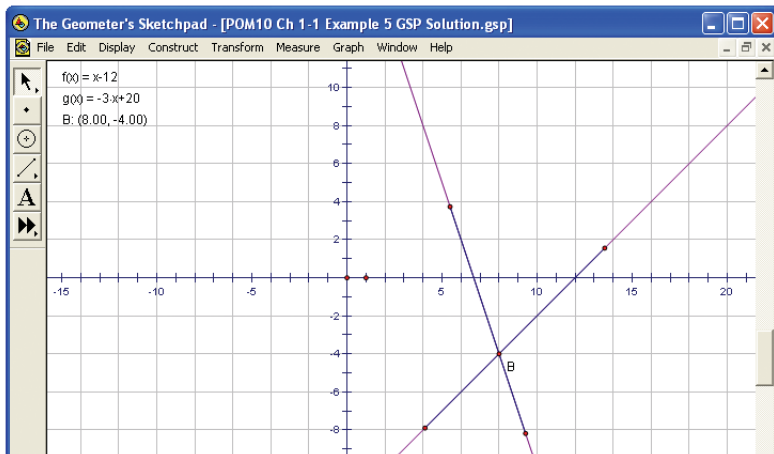
Method 2: Use *The Geometer's Sketchpad*®

Open *The Geometer's Sketchpad*®. Choose **Show Grid** from the **Graph** menu. Drag the unit point until the workspace shows a grid up to 10 in each direction.

Choose **Plot New Function** from the **Graph** menu. The expression editor will appear. Enter the expression $x - 12$, and click **OK**. Repeat to plot the second function.

Note the location of the point of intersection of the two lines. Draw two points on each line, one on each side of the intersection point. Construct line segments to join each pair of points. Select the line segments. Choose **Intersection** from the **Construct** menu. Right-click on the point of intersection and select **Coordinates**. The coordinates of the point of intersection are displayed.

The point of intersection is $(8, -4)$.

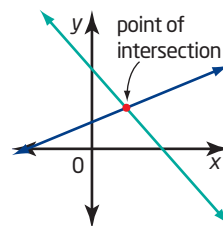


Making Connections

Refer to the Technology Appendix for help with *The Geometer's Sketchpad*® basics.

Key Concepts

- When changing from words into algebra, read each sentence carefully and think about what the words mean. Translate into mathematical expressions using letters and numbers and mathematical operations.
- There are many different word phrases that can represent the same mathematical expression.
- To solve a system of two linear equations means to find the point of intersection of the two lines.
- A system of linear equations can be solved by graphing both lines and using the graph to find the point where the two lines intersect.
- If the two lines do not cross at a grid mark, or if the equations involve decimals, you can use technology to graph the lines and then find the point of intersection.
- Check an answer by substituting it into the two original equations. If both sides of each equation have equal values, the solution is correct.



Communicate Your Understanding

- C1** Work with a partner. Make up at least eight sentences to be converted to mathematical equations. Exchange lists with another pair and translate the sentences into equations. As a group of four, discuss the answers and any difficulties.
- C2** In a group of three, use chart paper to list different phrases that can be represented by the same mathematical symbol or expression. Post the chart paper around the classroom as prompts.
- C3** Your friend missed today's class. She calls to find out what you learned. Explain, in your own words, what it means to solve a system of equations.
- C4** Will a linear system always have exactly one point of intersection? Explain your reasoning.
- C5** Describe in words how you would solve the linear system $y = 3x + 1$ and $y = -2x + 3$.



Practise

For help with questions 1 to 6, see Example 1.

- Translate each phrase into an algebraic expression.
 - seven less than twice a number
 - four more than half a value
 - a number decreased by six, times another number
 - a value increased by the fraction two thirds
- Translate each phrase into an algebraic expression.
 - twice a distance
 - twenty percent of a number
 - double a length
 - seven percent of a price
- Translate each sentence into an algebraic equation.
 - One fifth of a number, decreased by 17, is 41.
 - Twice a number, subtracted from five, is three more than seven times the number.
 - When tickets to a play cost \$5 each, the revenue at the box office is \$825.
 - The sum of the length and width of a backyard pool is 96 m.
- For each of the following, write a word or phrase that has the opposite meaning.
 - increased b) added
 - plus d) more than
- All of the words and phrases in question 4 are represented by the same operation in mathematics. What operation is it?
 - Work with a partner. Write four mathematical words or phrases for which there is an opposite. Trade your list with another pair in the class and give the opposites of the items in each other's list.

- Explain in your own words the difference between an expression and an equation. Explain how you can tell by reading whether words can be represented by an expression or by an equation. Provide your own examples.

For help with question 7, see Example 2.

- Which is the point of intersection of the lines $y = 3x + 1$ and $y = -2x + 6$?
A (0, 1) **B** (1, 1)
C (1, 4) **D** (2, 5)

For help with questions 8 and 9, see Example 3.

- Find the point of intersection for each pair of lines. Check your answers.
 - $y = 2x + 3$ **b)** $y = -x - 7$
 $y = 4x - 1$ $y = 3x + 5$
 - $y = \frac{1}{2}x - 2$ **d)** $y = 4x - 5$
 $y = \frac{3}{4}x + 3$ $y = \frac{2}{3}x + 5$
- Find the point of intersection for each pair of lines. Check your answers.
 - $x + 2y = 4$ **b)** $y + 2x = -5$
 $3x - 2y = 4$ $y - 3x = 5$
 - $3x - 2y = 12$ **d)** $x - y = 1$
 $2y - x = -8$ $x + 2y = 4$

For help with question 10, see Example 5.

- Use Technology** Use a graphing calculator or *The Geometer's Sketchpad*® to find the point of intersection for each pair of lines. Where necessary, round answers to the nearest hundredth.
 - $y = 7x - 23$ **b)** $y = -3x - 6$
 $y = -4x + 10$ $y = -6x - 20$
 - $y = 6x - 4$ **d)** $y = -3x + 4$
 $y = -5x + 12$ $y = 4x + 13$
 - $y = 5.3x + 8.5$ **f)** $y = -0.2x - 4.5$
 $y = -2.7x - 3.4$ $y = -4.8x + 1.3$

Connect and Apply

- 11.** Fitness Club CanFit charges a \$150 initial fee to join the club and a \$20 monthly fee. Fitness 'R' Us charges an initial fee of \$100 and \$30/month.
- Write an equation to represent the cost of membership at CanFit.
 - Write an equation to represent the cost of membership at Fitness 'R' Us.
 - Graph the two equations.
 - Find the point of intersection.
 - What does the point of intersection represent?
 - If you are planning to join for 1 year, which club should you join? Explain your answer.
- 12.** LC Video rents a game machine for \$10 and video games for \$3 each. Big Vid rents a game machine for \$7 and video games for \$4 each.
- Write a linear equation to represent the total cost of renting a game machine and some video games from LC Video.
 - Write a linear equation to represent the total cost of renting a game machine and some video games from Big Vid.
 - Find the point of intersection of the two lines from parts a) and b).
 - Explain what the point of intersection represents in this context.
- 13.** Jeff clears driveways in the winter to make some extra money. He charges \$15/h. Hesketh's Snow Removal charges \$150 for the season.
- Write an equation for the amount Jeff charges to clear a driveway for the season.
 - Write an equation for Hesketh's Snow Removal.
 - What is the intersection point of the two linear equations?
 - In the context of this question, what does the point of intersection represent?
- 14. Use Technology** Brooke is planning her wedding. She compares the cost of places to hold the reception.
- Limestone Hall: \$5000 plus \$75/guest
Frontenac Hall: \$7500 plus \$50/guest
- Write an equation for the cost of Limestone Hall.
 - Write an equation for the cost of Frontenac Hall.
 - Use a graphing calculator to find for what number of guests the hall charges are the same.
 - In what situation is Limestone Hall less expensive than Frontenac Hall? Explain.
 - What other factors might Brooke need to consider when choosing a banquet hall?
- 15. Use Technology** Gina works for a clothing designer. She is paid \$80/day plus \$1.50 for each pair of jeans she makes. Dexter also works for the designer, but he makes \$110/day and no extra money for finishing jeans.
- Write an equation to represent the amount that Gina earns in 1 day. Graph the equation.
 - Write an equation to represent the amount that Dexter earns in 1 day. Graph this equation on the same grid as in part a).
 - How many pairs of jeans must Gina make in order to make as much in a day as Dexter?
- 16.** Ramona has a total of \$5000 to invest. She puts part of it in an account paying 5%/year interest and the rest in a GIC paying 7.2% interest. If she has \$349 in simple interest at the end of the year, how much was invested at each rate?

- 17. Chapter Problem** The Clarke family called two car rental agencies and were given the following information.

Cool Car Company will rent them a luxury car for \$525 per week plus 20¢/km driven.

Classy Car Company will rent them the same type of car for \$500 per week plus 30¢/km driven.

- Let C represent the total cost, in dollars, and d represent the distance, in kilometres, driven by the family. Write an equation to represent the cost to rent from Cool Car Company.
- Write an equation to represent the cost to rent from Classy Car Company.
- Draw a graph to find the distance for which the cost is the same.
- Explain what your answer to part c) means in this context.

Extend

- 18.** Alain has just obtained his flight instructor's rating. He is offered three possible pay packages at a flight school.
- a flat salary of \$25 000 per year
 - \$40/h of instruction for a maximum of 25 h/week for 50 weeks
 - \$300/week for 50 weeks, plus \$25/h of instruction for a maximum of 25 h/week
- For each compensation package, write an equation that models the earnings, E , in terms of the number of hours of instruction, n .
 - Graph each equation, keeping in mind the restrictions on the flying hours.
 - Use your graph to write a note of advice to Alain about which package he should take, based on how many hours of instruction he can expect to give.

- 19.** Graph the equations $3x - y + 1 = 0$, $y = 4$, and $2x + y - 6 = 0$ on the same grid. Explain what you find.

- 20. a)** Can you solve the linear system $y = 2x - 3$ and $4x - 2y = 6$? Explain your reasoning.
- b)** Can you solve the linear system $y = 2x - 3$ and $4x - 2y = 8$? Explain your reasoning.
- c)** Explain how you can tell, without solving, how many solutions a linear system has.

- 21.** Solve the following system of equations by graphing. How is this system different from the ones you have worked with in this section?

$$y = x - 4$$
$$y = -x^2 + x$$

- 22. Math Contest** A group of 15 explorers and two children come to a crocodile-infested river. There is a small boat, which can hold either one adult or two children.
- How many trips must the boat make across the river to get everyone to the other side?
 - Write a formula for the number of trips to get n explorers and two children across the river.
- 23. Math Contest** A number is called *cute* if it has four different whole number factors. What percent of the first twenty-five whole numbers are cute?

- 24. Math Contest** The average of 13 consecutive integers is 162. What is the greatest of these integers?

A 162 **B** 165 **C** 168 **D** 172 **E** 175