

1.3

Investigate Equivalent Linear Relations and Equivalent Linear Systems

In these investigations, use the most convenient method to graph each linear relation:

- a table of values
- slope and y -intercept
- x - and y -intercepts
- a graphing calculator
- *The Geometer's Sketchpad*®



Tools

- grid paper, graphing calculators, or geometry software

equivalent linear equations

- equations that have the same graph

Investigate A

What are equivalent linear equations?

- a) On the same grid, graph the lines $x + 2y = 4$ and $2x + 4y = 8$.
 - b) How are the graphs related?
 - c) How are the equations related?
- a) On the same grid, graph the lines $y = -\frac{1}{2}x + 3$ and $x + 2y = 6$.
 - b) How are the graphs related?
 - c) How are the equations related?
- a) Without graphing, tell which two of the following are **equivalent linear equations**.
 $y - x + 5 = 0$ $y = 3x + 15$ $2y = 2x - 10$
 - b) Check your answer by graphing the three equations.
- Which one of the following is *not* equivalent to the others?
 $2x - 4y = 8$ $y = \frac{1}{2}x - 2$ $2y - x - 4 = 0$
- Write two equivalent equations for each of the following. Check by graphing.
 - a) $3x + 2y = 12$
 - b) $x + y = 4$
 - c) $y = \frac{2}{3}x + 1$
- Reflect**
 - a) Describe how to obtain an equivalent equation for any linear relation.
 - b) How many equivalent equations are there for a given linear relation?



Tools

- grid paper, graphing calculators, or geometry software

equivalent linear systems

- pairs of linear equations that have the same point of intersection

Investigate B

What are equivalent linear systems?

- Graph the linear system and find the point of intersection.

$$y = x - 1$$

$$y = -\frac{1}{2}x + 2$$

- Graph the linear system and find the point of intersection.

$$2x - 2y - 2 = 0$$

$$2y + x = 4$$

- Compare the solutions to questions 1 and 2. What do you notice?

- Compare the equations in questions 1 and 2. How are the equations related?

- Graph the linear system and find the point of intersection.

$$y = 2x + 1$$

$$y + x = 7$$

- Choose a number. Multiply the first equation in part a) by the number. How is the new equation related to the first equation in part a)?

- Choose another number. Multiply the second equation in part a) by the number. How is the new equation related to the second equation in part a)?

- If you graphed the two new equations that you obtained in parts b) and c), what would you expect the point of intersection to be? Explain why. Check by graphing.

- Reflect** Explain how you can use equivalent linear equations to write an **equivalent linear system**. Use your own examples in your explanation.

- Graph the linear system and find the point of intersection.

$$x + 2y = 4$$

$$x - y = 1$$

- If you add the left sides and the right sides of the two equations in part a), you obtain the equation $2x + y = 5$. Graph this equation on the same grid as in part a). What do you find?

- If you subtract the left sides and the right sides of the two equations in part a), you obtain the equation $3y = 3$. Graph this equation on the same grid as in part a). What do you find?

7. a) Graph the linear system and find the point of intersection.

$$2x + 3y = -4$$

$$x + 2y = -3$$

- b) On the same coordinate grid, graph the equation $3x + 5y = -7$. What do you notice? How is this equation related to the two equations in part a)?
- c) On the same coordinate grid, graph the equation $x + y = -1$. What do you notice? How is this equation related to the two equations in part a)?
8. a) Graph the linear system and find the point of intersection.
- $$3x - 2y = 18 \quad \textcircled{1}$$
- $$2x + y = 12 \quad \textcircled{2}$$
- b) Obtain a new equation, $\textcircled{3}$, by adding the left sides and the right sides of the equations in part a). If you graphed the linear system formed by equations $\textcircled{1}$ and $\textcircled{3}$, what result would you expect? Check by graphing.
- c) If you graphed the linear system formed by equations $\textcircled{2}$ and $\textcircled{3}$, what result would you expect? Check by graphing.
- d) Obtain a new equation, $\textcircled{4}$, by subtracting the left sides and the right sides of the equations in part a). If you graphed the linear system formed by equations $\textcircled{1}$ and $\textcircled{4}$, or by equations $\textcircled{2}$ and $\textcircled{4}$, what result would you expect? Check by graphing.
- e) Do you think the linear system formed by equations $\textcircled{3}$ and $\textcircled{4}$ will give the same result? Check by graphing.
9. **Reflect** Given a linear system of two equations in two variables, describe at least three ways in which you can obtain an equivalent linear system. Provide your own examples to illustrate.

Key Concepts

- Equivalent linear equations have the same graph.
- For any linear equation, an equivalent linear equation can be written by multiplying the equation by any real number.
- Equivalent linear systems have the same solution. The graphs of the linear relations in the system have the same point of intersection.
- Equivalent linear systems can be written by writing equivalent linear equations for either or both of the equations, or by adding or subtracting the original equations.

Communicate Your Understanding

- C1** Rohan claims that the following linear equations will have the same graph. Is he correct? Explain why or why not.

$$y = \frac{3}{4}x + 1 \text{ and } 4y = 3x + 4$$

- C2** If $y = 2x - 5$ and $3y = kx - 15$ are equivalent linear equations, what is the value of k ?

- C3** Are the linear systems A and B equivalent? Explain how you can tell from the equations. How could you check using a graph?

System A

$$y = 2x - 2$$

$$y = x + 1$$

System B

$$y = 2x - 2$$

$$2y = 2x + 2$$

- C4** The graph of the following linear system is shown.

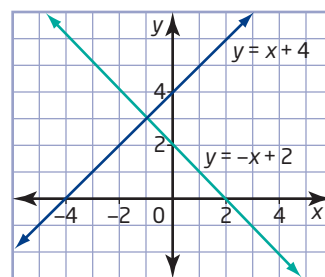
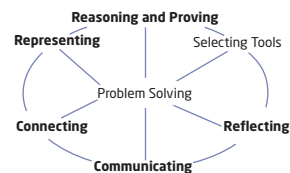
$$y = x + 4 \quad \textcircled{1}$$

$$y = -x + 2 \quad \textcircled{2}$$

The following is an equivalent linear system. Explain how you can tell from the graph. How can these equations be obtained from equations $\textcircled{1}$ and $\textcircled{2}$?

$$x = -1$$

$$y = 3$$



Practise

1. Which two equations are equivalent?

A $y = \frac{1}{2}x + 3$

B $y = x + 6$

C $2y = x + 6$

2. Which is *not* an equivalent linear relation?

A $8y = 12x + 4$

B $4y = 6x + 2$

C $2y = 3x + 4$

D $y = \frac{3}{2}x + \frac{1}{2}$

3. Write two equivalent equations for each.

a) $y = 3x - 2$

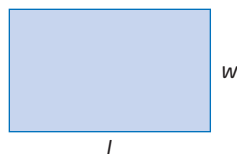
b) $3x + 6y = 12$

c) $y = \frac{3}{5}x + 2$

d) $8x + 4y = 10$

Connect and Apply

4. The perimeter of the rectangle is 24. Write an equation to represent this situation. Then, write an equivalent linear equation.



5. The value of the nickels and dimes in Tina's wallet is 70¢. Write an equation to represent this information. Then, write an equivalent linear equation.

6. A linear system is given.

$$3x - 6y = 15 \quad \textcircled{1}$$

$$x + y = 3 \quad \textcircled{2}$$

Explain why the following is an equivalent linear system.

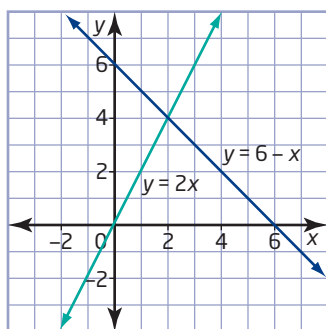
$$x - 2y = 5$$

$$2x + 2y = 6$$

7. A linear system is shown on the graph.

$$y = 2x \quad \textcircled{1}$$

$$y = 6 - x \quad \textcircled{2}$$



- a) Use a graph to show that the following is an equivalent linear system.

$$2y = x + 6 \quad \textcircled{3}$$

$$0 = 3x - 6 \quad \textcircled{4}$$

- b) How is equation $\textcircled{3}$ obtained from equations $\textcircled{1}$ and $\textcircled{2}$?
 c) How is equation $\textcircled{4}$ obtained from equations $\textcircled{1}$ and $\textcircled{2}$?

8. A linear system is given.

$$y = \frac{2}{3}x - 1 \quad \textcircled{1}$$

$$y = -\frac{1}{3}x + 2 \quad \textcircled{2}$$

- a) Explain why the following is an equivalent linear system.

$$3y - 2x = -3 \quad \textcircled{3}$$

$$3y + x = 6 \quad \textcircled{4}$$

- b) If you graph the four equations, what result do you expect? Graph to check.

Extend

9. Work backward to build a more complicated linear system. Start with a solution, for example $x = 3$ and $y = -2$. Choose your own example.

- Write an equivalent linear system by adding and then subtracting the two equations in your solution.
- Multiply each equation from part a) by a different number to write another equivalent system.
- Use other ways to write equivalent linear equations to transform your linear system.
- Use graphing or substitution to check that your result in part c) has the same solution as you started with.
- Exchange your linear system from part c) with that of another student. Solve the linear system.

10. **Math Contest** The self-taught Indian mathematician Srinvasa Ramanujan (1887–1920) discovered more than 3000 theorems. One of his challenge problems was to find the least number that can be written as the sum of two cubes in two different ways. Find the number.

11. **Math Contest** If the two spinners shown are each spun once, what is the probability that the sum of the two numbers is either even or a multiple of 3?

A $\frac{7}{13}$ B $\frac{2}{3}$ C $\frac{1}{3}$ D $\frac{9}{13}$ E $\frac{3}{4}$

