1.4 The Method of Elimination

You have now seen how to solve a linear system by graphing or by substitution. There is another algebraic method as well. With each new method, you have more options for solving the linear system.

Investigate

How can you solve a linear system by elimination?

Parnika and her mother, Mati, share a digital camera. They use two memory cards to store the photos. While on vacation, they took a total of 117 photos. There are 41 more photos on Parnika's memory card than on her mother's.

1. Read the situation described above. Let p represent the number of photos on Parnika's memory card and m represent the number of photos on Mati's memory card.

a) Write an equation to represent the total number of photos on the memory cards.

b) Write an equation to represent the difference in the number of photos on the memory cards.

2. a) Write your two equations below one another, so like terms align in columns. Add like terms on the left sides and add the right sides of the equations.

b) Which variable has disappeared?

c) Solve for the remaining variable.

d) Substitute your answer from part c) into the first equation. Solve for the other variable.

e) How many photos are on Parnika's memory card? on Mati's memory card?

3. a) Write the pair of equations from step 1 again. Put a line under the two equations and subtract the bottom equation from the top equation.

b) Which variable has disappeared?

c) Solve for the remaining variable.

d) Substitute your answer from part c) into the first equation. Solve for the other variable.

e) How many photos are on each person's memory card?

4. **Reflect**

a) Explain what you have done in order to find the number of photos on the memory cards.

b) How can you verify that you have obtained the correct solution?

In the Investigate above you solved a linear system by the **method of elimination**. This is another method for solving a system of linear equations.

method of elimination

- solving a linear system by adding or subtracting to eliminate one of the variables

Example 1 Solve a Linear System Using the Method of Elimination

Solve the system of linear equations.

$3x + y = 19$

$4x - y = 2$

Check your solution.

Solution

$3x + y = 19$ ①

$4x - y = 2$ ②

$7x = 21$ ① + ②

Add columns vertically.

I notice that I have $+y$ in the first equation and $-y$ in the second equation. If I add the two equations, y will be eliminated.

$x = \frac{21}{7}$

$x = 3$

Now I have one equation in one variable. I can solve for x.

Substitute $x = 3$ into equation ① to find the corresponding y-value.

$3x + y = 19$

$3(3) + y = 19$

$9 + y = 19$

$y = 10$

I can substitute back into either original equation.

Check by substituting $x = 3$ and $y = 10$ into both original equations.

In $3x + y = 19$:

L.S. $= 3x + y$ **R.S.** $= 19$

$= 3(3) + 10$

$= 19$

L.S. $=$ **R.S.**

In $4x - y = 2$:

L.S. $= 4x - y$ **R.S.** $= 2$

$= 4(3) - 10$

$= 2$

L.S. $=$ **R.S.**

The solution checks in both equations.

The solution to the linear system is $x = 3$ and $y = 10$.

Example 2 Solve Using Elimination

Solve the linear system.

$10x + 4y = -1$
$8x - 2y = 7$

Solution

$$10x + 4y = -1 \quad ①$$
$$8x - 2y = 7 \quad ②$$

①	$10x + 4y = -1$
$2 \times$ ②	$16x - 4y = 14$
① $+ 2 \times$ ②	$26x = 13$
	$x = \frac{13}{26}$
	$x = \frac{1}{2}$

I can't eliminate either variable by adding or subtracting the equations given. If I multiply equation ② by 2, then I will have $-4y$ in the second equation. Then, I can add to eliminate the y-terms.

Substitute $x = \frac{1}{2}$ in ② to find the corresponding y-value.

$$\begin{aligned} 8x - 2y &= 7 \\ 8\left(\frac{1}{2}\right) - 2y &= 7 \\ 4 - 2y &= 7 \\ -2y &= 7 - 4 \\ -2y &= 3 \\ y &= \frac{3}{-2} \\ y &= -\frac{3}{2} \end{aligned}$$

I chose to substitute in ② because that equation looks simpler.

Check: Substitute $x = \frac{1}{2}$ and $y = -\frac{3}{2}$ into both original equations.

In $10x + 4y = -1$:

L.S. $= 10x + 4y$ **R.S.** $= -1$

$= 10\left(\frac{1}{2}\right) + 4\left(-\frac{3}{2}\right)$

$= 5 - 6$

$= -1$

L.S. = R.S.

In $8x - 2y = 7$:

L.S. $= 8x - 2y$ **R.S.** $= 7$

$= 8\left(\frac{1}{2}\right) - 2\left(-\frac{3}{2}\right)$

$= 4 + 3$

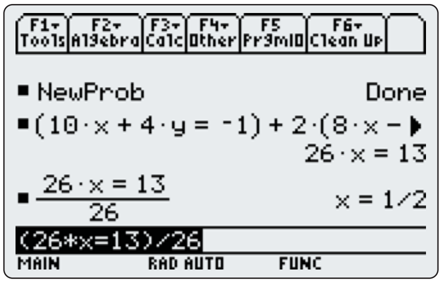
$= 7$

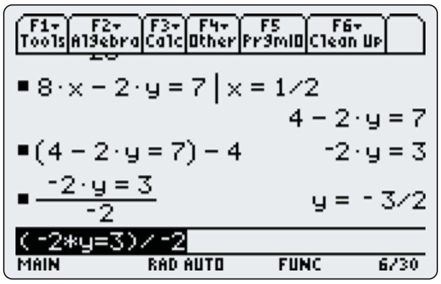
L.S. = R.S.

The solution to the linear system is $x = \frac{1}{2}$, $y = -\frac{3}{2}$.

You can check your work using a Computer Algebra System (CAS).

- Type in equation ①, in brackets.
- Add equation ②, in brackets, multiplied by 2.
- Press ENTER.
- Divide the resulting equation by 26.
- Substitute $x = \frac{1}{2}$ in equation ① and solve for *y*.





Example 3 Find a Point of Intersection Using Elimination

Find the point of intersection of the linear system.

$4x + 3y = 13$

$5x - 4y = -7$

Verify your answer.

Solution

$4x + 3y = 13$ ①

$5x - 4y = -7$ ②

I'll need to multiply each of the equations to get the same coefficient in front of one of the variables. If I multiply equation ① by 5 and equation ② by 4, both equations will start with 20*x*.

Method 1: Eliminate *x*

$$\begin{aligned}
5 \times ① \quad & 20x + 15y = 65 \\
4 \times ② \quad & 20x - 16y = -28 \\
& 31y = 93 \\
& y = \frac{93}{31} \\
& y = 3
\end{aligned}$$

Now if I subtract, *x* will be eliminated. In the *y*-column, $15y - (-16y) = 15y + 16y$. On the right, $65 - (-28) = 65 + 28$.

Substitute $y = 3$ into ① to find the corresponding *x*-value.

$$\begin{aligned}
4x + 3y &= 13 \\
4x + 3(3) &= 13 \\
4x + 9 &= 13 \\
4x &= 4 \\
x &= 1
\end{aligned}$$

Method 2: Eliminate *y*

$4x + 3y = 13$ ①
$5x - 4y = -7$ ②

If I multiply ① by 4 and ② by 3, one equation will have 12*y* and the other will have −12*y*. Then, if I add, *y* will be eliminated.

$$\begin{aligned} 4 \times ① \quad & 16x + 12y = 52 \\ 3 \times ② \quad & 15x - 12y = -21 \\ & 31x = 31 \\ & x = 1 \end{aligned}$$

Substitute $x = 1$ into ① to find the corresponding y-value.

$$\begin{aligned} 4x + 3y &= 13 \\ 4(1) + 3y &= 13 \\ 4 + 3y &= 13 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

Verify by substituting $x = 1$ and $y = 3$ into both original equations.

In $4x + 3y = 13$:

L.S. $= 4x + 3y$ **R.S.** $= 13$
$= 4(1) + 3(3)$
$= 4 + 9$
$= 13$
L.S. = R.S.

In $5x - 4y = -7$:

L.S. $= 5x - 4y$ **R.S.** $= -7$
$= 5(1) - 4(3)$
$= 5 - 12$
$= -7$
L.S. = R.S.

The point of intersection of the lines is (1, 3).

Example 4 Solve a Problem Using the Method of Elimination

A small store sells used CDs and DVDs. The CDs sell for \$9 each. The DVDs sell for \$11 each. Cody is working part time and sells a total of \$204 worth of CDs and DVDs during his shift. He knows that 20 items were sold. He needs to tell the store owner how many of each type were sold. How many CDs did Cody sell? How many DVDs did Cody sell?

Solution

Let c represent the number of CDs sold.
Let d represent the number of DVDs sold.

$c + d = 20$ ①
$9c + 11d = 204$ ②

The number of CDs plus the number of DVDs is 20.

\$9 for each CD plus \$11 for each DVD totals \$204.

Multiply ① by 9.

$$\begin{aligned} 9c + 9d &= 180 \\ 9c + 11d &= 204 \\ -2d &= -24 \\ d &= 12 \end{aligned}$$

I can also solve this system using substitution or graphing.

If I subtract, *c* is eliminated.

Substitute $d = 12$ into one of the original equations to solve for c.

$$\begin{aligned} c + d &= 20 \\ c + 12 &= 20 \\ c &= 8 \end{aligned}$$

Check in the original word problem:
Money: 8 CDs at \$9 is \$72, and 12 DVDs at \$11 is \$132. The total is \$204.
Number of items: 8 CDs and 12 DVDs is 20 items sold.

Cody sold eight CDs and twelve DVDs during his shift.

Key Concepts

- To solve a linear system by elimination, follow these steps:
  - Arrange the two equations so that like terms are aligned.
  - Choose the variable you wish to eliminate.
  - If necessary, multiply one or both equations by a value so that they have the same or opposite coefficient in front of the variable you want to eliminate.
  - Add or subtract (as needed) to eliminate one variable.
  - Solve for the remaining variable.
  - Substitute into one of the original equations to find the value of the other variable.
  - Check your solution by substituting into the original equations, or into the word problem.
  - If you are solving a word problem, write the answer in words.

Communicate Your Understanding

C1 Consider solving the linear system $\begin{aligned} x + y &= 5 \\ x - y &= 7 \end{aligned}$

a) To eliminate x, do you add or subtract the two equations?

b) To eliminate y, do you add or subtract the two equations?

c) Will you obtain the same solution if you add or subtract the two equations? Explain.

C2 Consider solving the linear system $\begin{aligned} 4x + 3y &= 15 \quad ① \\ 8x - 9y &= 15 \quad ② \end{aligned}$

a) Describe the steps you would use to eliminate x.

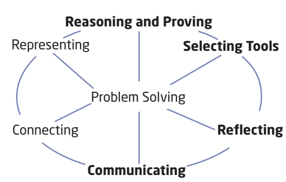
b) The linear system can also be solved by first eliminating y. Describe the steps you would use if you chose this method.

C3 In what situations would you use the method of graphing? substitution? elimination? Consider the following linear systems. Which method would you use for each and why?

a) $y = x - 9$
$2x + 3y = 1$

b) $3x + 2y = 8$
$2x - 2y = 7$

c) $y = -\frac{2}{3}x + 5$
$3x - 2y = 6$



Practise

For help with questions 1 and 2, see Example 1.

1. Solve using the method of elimination.

a) $x + y = 2$
$3x - y = 2$

b) $x - y = -1$
$3x + y = -7$

c) $x + 3y = 7$
$x + y = 3$

d) $5x + 2y = -11$
$3x + 2y = -9$

2. Solve using the method of elimination. Check each solution.

a) $2x + y = -5$
$-2x + y = -1$

b) $4x - y = -1$
$-4x - 3y = -19$

c) $2x + y = 8$
$4x - y = 4$

d) $3x + 2y = -1$
$-3x + 4y = 7$

For help with questions 3 and 4, see Example 2.

3. Find the point of intersection of each pair of lines.

a) $x + 2y = 2$
$3x + 5y = 4$

b) $3x + 5y = 12$
$2x - y = -5$

c) $3x + y = 13$
$2x + 3y = 18$

d) $6x + 5y = 12$
$3x - 4y = 6$

4. Solve by elimination. Check each solution.

a) $4x + 3y = 4$
$8x - y = 1$

b) $5x - 3y = 25$
$10x + 3y = 5$

c) $5x + 2y = 48$
$x + y = 15$

d) $2x + 3y = 8$
$x - 2y = -3$

For help with questions 5 to 7, see Example 3.

5. Solve by elimination. Check each solution.

a) $3x - 2y = 5$
$2x + 3y = 12$

b) $5m + 2n = 5$
$2m + 3n = 13$

c) $3a - 4b = 10$
$5a - 12b = 6$

d) $3h - 4k = 5$
$5h + 3k = -11$

6. Find the point of intersection of each pair of lines. Check each solution.

a) $3x + y = 13$
$2x + 3y = 18$

b) $2x + 3y = -18$
$3x - 5y = 11$

c) $3x - 2y + 2 = 0$
$7x - 6y + 11 = 0$

d) $2a - 3b = -10$
$4a + b = 1$

7. Solve each system of linear equations by elimination. Check your answers.

a) $4x - 9y = 4$
$6x + 15y = -13$

b) $2x + 9y = -4$
$5x - 2y = 39$

c) $3a - 2b + 4 = 0$
$2a - 5b - 1 = 0$

d) $2u + 5v = 46$
$3u - 2v = 12$

Connect and Apply

For help with questions 8 and 9, see Example 4.

8. Mehrab works in a department store selling sports equipment. Baseball gloves cost \$29 each and bats cost \$14 each. One shift, he sells 28 items. His receipts total \$647.

a) How many bats did Mehrab sell?

b) How many gloves did he sell?

9. Liz works at the ballpark selling bottled water. She sells 37 bottles in one shift. The large bottles sell for \$5 each and the small bottles sell for \$3 each. At the end of one game, she has taken in \$131.

a) How many large bottles did Liz sell?

b) How many small bottles did she sell?

10. Consider the linear system $2x - 3y = 5$ and $4x + y = 8$.

a) Solve by elimination.

b) Solve by substitution.

c) Which method do you prefer? Why?

11. Explain how you would solve the system $3x + 2y = 5$ and $4x + 5y = 11$ using the method of elimination. Do not actually solve the system.

12. Expand and simplify each equation. Then, solve the linear system.

a) $2(3x - 1) - (y + 4) = -7$
$4(1 - 2x) - 3(3 - y) = -12$

b) $3(a - 1) - 3(b - 3) = 0$
$3(a + 2) - (b - 7) = 20$

c) $5(k + 5) - 2(n - 3) = 62$
$4(k - 7) - (n + 4) = -9$

13. To solve the following linear system by elimination, Brent first multiplied each equation by 10. Explain why he did this step. Complete the solution.

$0.3x - 0.5y = 1.2$
$0.7x - 0.2y = -0.1$

14. Solve each linear system.

a) $0.2x - 0.3y = 1.3$
$0.5x + 0.2y = 2.3$

b) $0.1a - 0.4b = 1.9$
$0.4a + 0.5b = -0.8$

15. Bhargav stops in at a deli to get lunch for his crew. He buys five roast beef and three vegetarian sandwiches and the order costs \$27.50. The next week, he pays \$23.00 for two roast beef and six vegetarian sandwiches. How much does one roast beef sandwich cost?

16. Maria rented the same car twice in one month. She paid \$180 the first time for 3 days and she drove a total of 150 km. The next time, she also paid \$180 and had the vehicle for only 2 days, but travelled 400 km.

a) What was the cost per day?

b) What was the cost per kilometre?

17. Chapter Problem The Clarke's son suggests that they rent a car that costs \$250 for the week plus 22¢/km. Their daughter does not want to drive far, so she suggests a car that is only \$96 for the week but 50¢/km.

a) Write an equation to represent the cost of the car suggested by the son.

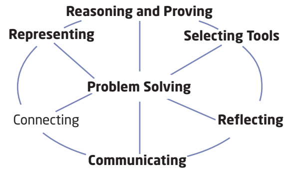
b) Write an equation to represent the cost of the car suggested by the daughter.

c) When will the two cars cost the same? Use the method of elimination to solve.

d) If the Clarkes plan to drive 500 km, which option is less expensive?

18. What happens when you solve the system $2x + 3y = 6$ and $6x + 9y = 0$ by elimination? Use a graph in your explanation.

Achievement Check



19. a) Nita's class visited a provincial site to view some ancient rock drawings. Two adults and five students in one van paid \$77 for the visit. Two adults and seven students in a second van paid \$95. What were the entry prices for a student and an adult? Verify your solution.

b) Katie and Chris each solved a system of two linear equations as shown. Whose method is correct? Explain why.

Katie

$2x + y = 5$
$+ \ x - y = 1$
$3x = 6$
$x = 2$
$2x + y = 5$
$2(2) + y = 5$
$4 + y = 5$
$y = 1$
The solution is (2, 1).

Chris

$2x + y = 5$
$- \ x - y = 1$
$x = 4$
$4 - y = 1$
$-y = -3$
$y = 3$
The solution is (4, 3).

Extend

20. Solve by elimination.

a) $\frac{1}{2}m + n = -4$
$\frac{m}{2} - \frac{3n}{2} = 1$

b) $\frac{4a}{3} - \frac{b}{4} = 6$
$\frac{5a}{6} + b = 13$

c) $\frac{t-5}{3} + \frac{w+1}{2} = 1$
$\frac{t-1}{5} + \frac{w+2}{3} = 2$

21. Consider the linear system $ax + by = c$
$dx + ey = f$

Find a general solution for x and y. State any restrictions on the values of a, b, c, d, e, and f.

22. Solve the system of equations.

$x + 3y - z = -14$
$7x + 6y + z = 1$
$4x - 2y - 5z = 11$