1.5 Solve Problems Using Linear Systems

Now you know a number of different ways to solve a system of linear equations. You can solve

- graphically by hand
- graphically with a graphing calculator or graphing software
- algebraically by substitution
- algebraically by elimination

In this section, you will look at how to choose among these methods. You will also see how to apply the methods to some more challenging problems.

Investigate

Tools

- grid paper
- ruler

How do you choose a method for solving a linear system?

1. a) Graph the line $y = 3x + 1$.

b) On the same set of axes, graph the line $y = 4x - 3$.

c) What is the point of intersection of these two lines?

2. a) Graph the line $x + y = 101$.

b) On the same set of axes, graph the line $300x - y = 200$.

c) What is the point of intersection of these two lines?

3. Why was it easier to find the point of intersection of the two lines in step 1 than in step 2?

4. a) Use the method of substitution to find the intersection point of the lines $y = 3x + 1$ and $y = 4x - 3$.

b) Did you get the same result you found in step 1 part c)?

5. a) Use the method of elimination to find the intersection point of the lines $y = 3x + 1$ and $y = 4x - 3$.

b) Did you get the same result you found in step 1 part c)?

6. a) Find the solution to the linear system $x + y = 101$ and $300x - y = 200$ by substitution.

b) Did you get the same result you found in step 2 part c)?

7. a) Find the solution to the linear system $x + y = 101$ and $300x - y = 200$ by elimination.

b) Did you get the same result you found in step 2 part c)?

8. You have learned three methods for solving a linear system: graphing, substitution, and elimination.

a) Which method was easiest to use for the lines $y = 3x + 1$ and $y = 4x - 3$? Explain.

b) Which method was easiest to use for the lines $x + y = 101$ and $300x - y = 200$? Explain.

9. Reflect Consider pairs of equations that form a linear system.

a) Describe the equations in a linear system that you would choose to solve by graphing.

b) Describe the equations in a linear system that you would choose to solve by substitution.

c) Describe the equations in a linear system that you would choose to solve by elimination.

Example 1 Graphing, Substitution, or Elimination?

Christian has a total of eight cars and trucks to play with. His birthday is soon. He hopes to double the number of cars he has now. If he does, he will have a total of 11 cars and trucks. How many cars does he have now? How many trucks?

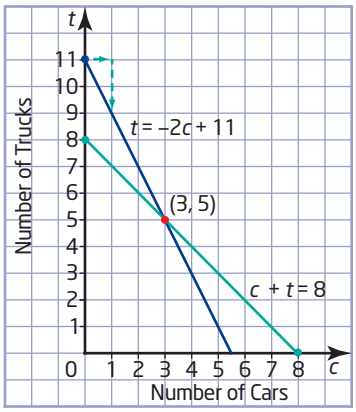
Solution

Let c represent the number of cars Christian has now.
Let t represent the number of trucks he has now.
$c + t = 8$
$2c + t = 11$

It doesn't make sense to have part of a car, so I expect whole-number answers. I will graph both equations. I'll put c on the horizontal axis and t on the vertical axis.

For the line $c + t = 8$, the intercepts are at (8, 0) and (0, 8).
Rearrange the second equation as $t = -2c + 11$.
The t-intercept is 11 and the slope is -2.

The solution is $c = 3$, $t = 5$.



Check in the problem:

Christian's cars and trucks now:
3 cars + 5 trucks = 8 toys

Christian's cars and trucks after his birthday: 6 cars + 5 trucks = 11 toys
Also, 6 cars is double 3 cars.
This checks.
Christian has three cars and five trucks now.

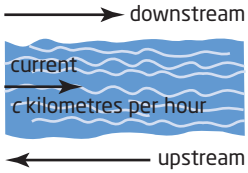
The problem in Example 1 was solved by graphing, but it can be solved by any of the three methods: graphing, substitution, or elimination.

Example 2 Solve a Distance, Speed, Time Problem

A canoeist took 2 h to travel 12 km down a river. The return trip, against the current, took 3 h. What was the average paddling rate of the canoeist? What was the speed of the current?

Solution

Let p represent the canoeist's average paddling speed, in kilometres per hour. Let c represent the speed of the current, in kilometres per hour. Draw a diagram to model the situation. Then, use a table to organize the given facts.



Going downstream, the current helps the canoeist. Going upstream, the current slows the canoeist down.

Direction	Distance (km)	Speed (km/h)	Time (h)
Downstream	12	$p + c$	2
Upstream	12	$p - c$	3

To write the equations, use the fact that distance = speed × time.

$12 = (p + c)2$ ①
$12 = (p - c)3$ ②

I can simplify each equation by dividing both sides of the first equation by 2, and both sides of the second equation by 3.

$6 = p + c$
$4 = p - c$
$10 = 2p$
$p = 5$

I can solve this linear system directly using elimination. I will add.

Substitute $p = 5$ into equation ① to find c.
$12 = (5 + c)2$
$12 = 10 + 2c$
$2 = 2c$
$c = 1$

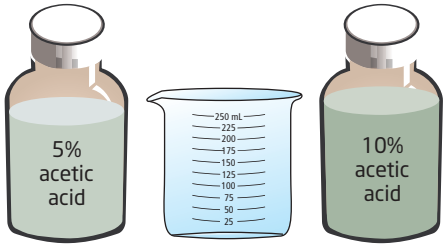
Verify in the original problem:
Downstream: Speed is $5 + 1$, or 6 km/h. So, in 2 h the distance is 12 km. This checks with the first sentence.
Upstream: Speed is $5 - 1$, or 4 km/h. So, in 3 h the distance is 12 km. This checks with the second sentence.
The canoeist's average paddling rate was 5 km/h. The speed of the current was 1 km/h.

Example 3 Solve a Mixture Problem

Marryam has a bottle of 5% acetic acid and a bottle of 10% acetic acid. How much of each should she use to make 250 mL of 8% acetic acid?

Solution

Let f represent the amount of 5% acid in the 8% mixture.
Let t represent the amount of 10% acid in the 8% mixture.



Use a table to organize the given information.

Volume (mL)	5% Acid	10% Acid	8% Mixture
Solution	f	t	250
Pure Acid	$0.05f$	$0.1t$	$0.08(250)$

$f + t = 250$ ①
$0.05f + 0.1t = 20$ ②

The sum of the two volumes is 250 mL.

These equations would not be easy to solve by graphing, unless I used a graphing calculator or multiplied equation ② by 20 first.

Consider the volume of pure acid. In the 8% mixture, 0.08(250) = 20.

Rearrange equation ①: $f = 250 - t$
Substitute into equation ②.

I'll use the method of substitution because it is easy to solve equation ① for one variable.

$$\begin{aligned} 0.05f + 0.1t &= 20 \\ 0.05(250 - t) + 0.1t &= 20 \\ 12.5 - 0.05t + 0.1t &= 20 \\ 12.5 + 0.05t &= 20 \\ 0.05t &= 7.5 \\ t &= \frac{7.5}{0.05} \\ t &= 150 \end{aligned}$$

Substitute $t = 150$ into equation ①.

$$\begin{aligned} f + t &= 250 \\ f + 150 &= 250 \\ f &= 100 \end{aligned}$$

Marryam should mix 100 mL of the 5% acetic acid with 150 mL of the 10% acetic acid to make 250 mL of 10% acetic acid.

Did You Know?

Household white vinegar is 5% acetic acid. A 5% acetic acid solution means that 5% pure acid is mixed with 95% water. For example, 1 L of white vinegar contains 50 mL of pure acetic acid and 950 mL of water.

Key Concepts

- You can solve linear systems using any of the three methods: graphing, substitution, or elimination.
- Look at the equations carefully to see if there is an advantage to solving using a particular method.

Communicate Your Understanding

C1 In what situations would solving by graphing be your preferred choice? Give an example.

C2 In what situations would solving by substitution be your preferred choice? Give an example.

C3 In what situations would solving by elimination be your preferred choice? Give an example.

C4 Write a linear system that can be solved by any of the three methods.

Practise

For help with questions 1 to 6, see Example 1.

1. Leanne works at a greenhouse. She needs to plant a total of 32 bulbs. Two types of bulbs are available. She is asked to plant three times as many crocus bulbs as tulip bulbs. How many of each should she plant?

2. James looks in his TV cabinet and finds some old Beta and VHS tapes. He has 17 tapes in all. He finds that he has three more Beta tapes than VHS tapes. How many of each type does he have?

3. The girls' soccer team held a fundraising car wash. They charged $5 for each car and $8 for each van. They washed 44 cars and vans and collected $262. How many of each type of vehicle did they wash?

4. Rehman invests his summer earnings of $3050. He invests part of the money at 8%/year, and the rest at 7.5%/year. After 1 year, these investments earn $242 in simple interest. How much did he invest at each rate?

5. Why might it be more appropriate to solve questions 1 and 2 by graphing than questions 3 and 4?

6. Consider the linear system

$3x - y = 8$
$4x - y = -15$

a) Which method would you choose to solve the linear system and why? Solve using the method you chose.

b) Now solve using one of the other methods available to you.

Connect and Apply

For help with questions 7 and 8, see Example 2.

7. Tyler rows 10 km downstream in 2 h. On the return trip, it takes him 4 h to travel 8 km. Determine his average rowing speed and the speed of the current.

8. With a tailwind, a plane flew the 3000 km from Calgary to Montréal in 5 h. The return flight, against the wind, took 6 h. Find the wind speed and the speed of the plane.

For help with questions 9 and 10, see Example 3.

9. Milk and cream contain different percents of butterfat. How much 3% milk needs to be mixed with how much 15% cream to give 20 L of 6% cream?

10. Amy needs to make 10 L of 42% sulphuric acid solution. In the supply room, she finds bottles of 30% sulphuric acid solution and 60% sulphuric acid solution. What volume of each solution should she mix in order to make the 42% solution?

11. To join Karate Klub, David must pay a monthly fee of $25 and an initial fee of $200. If he chooses Kool Karate, he must pay an initial fee of only $100 but $35/month.

a) After how many months is the cost the same at either karate club?

b) If David plans to try karate for 6 months, which club should he join?

c) If David decides to do karate for a year, which club should he join?

12. For a school band trip, Marcia decides to order T-shirts for all of the participants. It will cost $4 per shirt for the medium size, and $5 per shirt for the large size. Marcia orders a total of 70 T-shirts and spends $320. How many are medium shirts?

13. One type of granola has 30% nuts, by mass. A second type of granola has 15% nuts. What mass of each type needs to be mixed to make 600 g of granola that will have 21% nuts?

14. A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each should be used to make 500 g of an alloy that is 45% copper?

15. Some students at L.C.V.I. held a bake sale recently to raise money for a field trip. They charged $7 for fruit pies and $10 for meat pies. They sold a total of 52 pies and earned $424. How many of each type of pie did they sell?

16. A class trip is being planned. For one option, each student will pay $630. This includes two meals a day and accommodation for the 9-day trip. The other option offers three meals a day and accommodation for the 9 days. This second option costs $720. What is the cost per meal? What is the cost per day for accommodation?

Extend

17. Ian flew his airplane at best cruise speed for 2 h, then at economy cruise speed for 3 h, covering a total of 850 km. On the following day, he flew at best cruise speed for 3 h and at economy cruise speed for 2 h, covering a total of 900 km. Find the best cruise speed and the economy cruise speed for Ian's airplane.

18. A train leaves Toronto for Montréal at the same time as another train leaves Montréal for Toronto. The cities are 500 km apart. The trains pass each other 2 h later. The train from Montréal is travelling 50 km/h faster than the one from Toronto. At what distance away from Toronto do the trains pass each other?

19. Sam is a jewellery artist. She needs to mix metals to make her products. Pure gold is 24-karat and is very soft. It is usually mixed with other metals such as silver to make it harder. Sam has some 18-karat gold ($\frac{18}{24}$ pure gold) and some 9-karat gold ($\frac{9}{24}$ pure gold). What mass of each type of gold should she use to make 600 g of 15-karat gold?

20. **Math Contest** A chemist has one 30-L bottle of 15% hydrochloric acid and one 30-L bottle of 90% hydrochloric acid. She mixes 20 L of 60% hydrochloric acid and then pours 5 L of that solution back into the bottle containing the 90% hydrochloric acid. How strong is the acid in that bottle now?