4.1 Investigate Non-Linear Relations

non-linear relation

- a relationship between two variables that does not follow a straight line when graphed

Most relations that you have studied in mathematics have been linear. However, many **non-linear relations** also exist in real life. For example, the area of a shape is measured in square units, so the graph of area versus length is non-linear. Similarly, when a rocket is launched, it follows an arch-like path.

Investigate

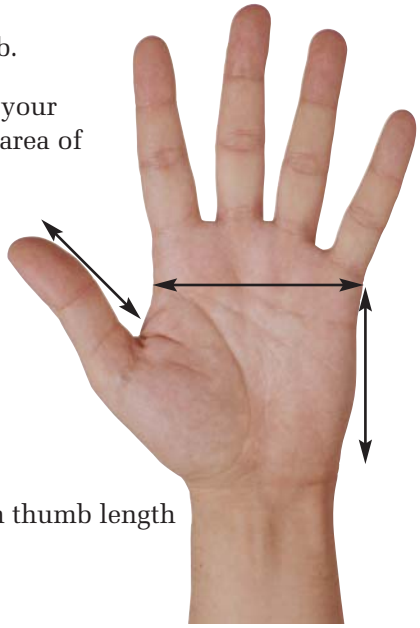
Tools

- ruler
- grid paper

How can you use a scatter plot to model non-linear data?

A: Relate Thumb Length and Palm Area

Work in small groups.



1. Measure the length of your thumb.
2. Measure the length and width of your palm. Calculate the approximate area of your palm.
3. Record the thumb length and palm area data for each group member.
4. Identify the independent and dependent variables.
5. Make a scatter plot of the data.
6. Describe the relationship between thumb length and palm area.
7. Draw a **curve of best fit**.
8. **Reflect** Why is a curve of best fit used for these data instead of a line of best fit?
9. Use your model to predict the area of a person's palm when that person's thumb is 8.1 cm long.

curve of best fit

- a smooth curve drawn to approximate the general path or trend in a scatter plot

B: Relate Distance and Roll Time

Work in small groups.

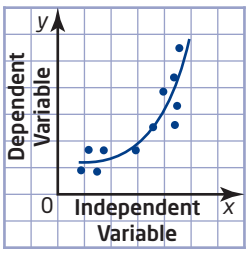
Tools

- board at least 1.2 m in length
- 2 textbooks
- 1 can of soup (or other object that rolls)
- metre stick or measuring tape
- stopwatch
- grid paper

1. Build a ramp using two textbooks as a support.
2. Place a can 30 cm from the bottom of the ramp.
3. Release the can and time how long it takes the can to roll to the bottom. You may wish to practise starting the stopwatch at the exact moment the can is released.
4. Repeat by releasing the can from 40 cm, 50 cm, and so on.
5. Record all your data in a table. Choose your variables.
6. Identify the independent and dependent variables. Explain your choices.
7. Make a scatter plot of the results.
8. Describe the relationship between distance and roll time.
9. Draw a curve of best fit.
10. **Reflect** Why is a curve of best fit used for these data instead of a line of best fit?
11. Use your model to predict the time it would take for a can to roll down a longer ramp with the same slope from a distance of 160 cm.

Key Concepts

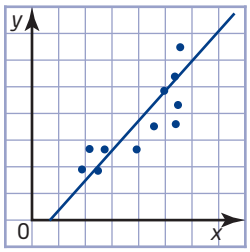
- The independent variable is the one that you control before the trial begins. The dependent variable is the one that you measure during the trial. It is affected by a change in the independent variable.
- Look at the pattern of the points in a scatter plot when deciding if the relation is linear or non-linear. The points in a non-linear relation will not lie along a line, but will form a graph that is curved.



Communicate Your Understanding

C1 State whether each line or curve of best fit is a good model for the data. Justify your answer.

a)



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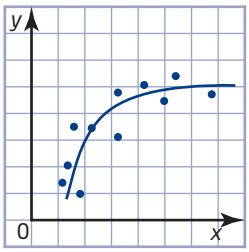
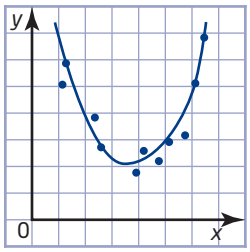
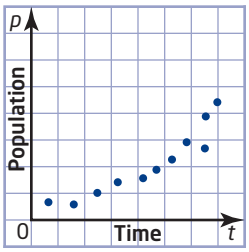


c)



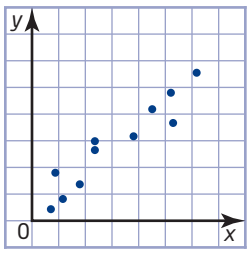
C2 The scatter plot shows the relationship between time, in 5-year intervals, and the population of a town. Explain why time was used as the independent variable.



Practise

1. Which scatter plot(s) could be modelled using a curve instead of a line of best fit? Explain.

a)



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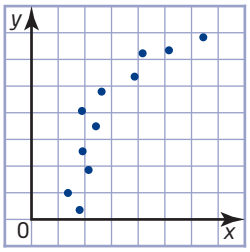
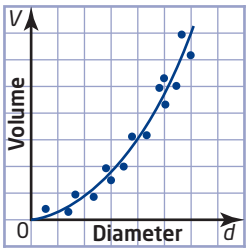


2. The scatter plot and curve of best fit show the relationship between the diameter of rain-collection barrels and the volume of water collected.

Is this relation linear or non-linear? Justify your answer.



3. An altimeter is attached to a model rocket before it is launched. The table shows the recorded data from the rocket's flight.

Time (s)	1	2	3	4	5	6	7
Height (m)	230	310	350	360	350	300	220

a) Make a scatter plot of the data.

b) Describe the relation.

c) Draw a curve of best fit.

d) Use your model to predict the height of the rocket after 8 s.

4. The table shows the average fuel economy of a car at a test track.

Speed (km/h)	Fuel Economy (L/100 km)
10	14.26
20	12.85
40	10.65
60	10.10
70	10.24
80	10.84
100	12.14
120	15.64
130	16.88
150	22.50

a) Make a scatter plot of the data.

b) Describe the relation.

c) Draw a curve of best fit.

d) Use your model to predict the fuel economy at 200 km/h.

e) This car does not get very good fuel economy. How would a graph of a car with better fuel economy look? Why?

Did You Know?

New vehicles have an EnerGuide label that shows the city and highway fuel-consumption ratings and an estimated annual fuel cost for that vehicle. The fuel-consumption ratings are provided by vehicle manufacturers and are based on standardized testing procedures and driving cycles performed under controlled conditions.

5. The table shows the data for a bouncing ball.

Bounce Number	1	2	3	4	5	6	7
Rebound Height (cm)	270	180	120	80	53	45	25

a) Make a scatter plot of the data.

b) Describe the relation.

c) Draw a curve of best fit.

d) How would the relationship change for a ball that was bouncier?

6. **Chapter Problem** A city opened a new landfill site in 2000. The table shows how much garbage was added to the landfill in each year from 2000 to 2007.

Year	Garbage Added (1000s of tonnes)
2000	200
2001	230
2002	258
2003	287
2004	317
2005	347
2006	376
2007	406

a) Determine the total mass of garbage in the landfill at the end of each year.

b) Make a scatter plot of the total mass of garbage versus the year. Draw a curve of best fit.

c) What problems do you predict if growth continues at its current rate?

7. A rectangle has a width of x centimetres, and its length is double its width.

a) Create a table comparing the length and area of a rectangle for widths up to 8 cm.

b) Make a scatter plot of the data.

c) Draw a curve of best fit.

d) Explain why the graph of this relation is non-linear.