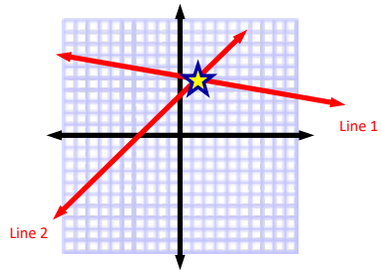
2.4 Types of Intersections & Equivalent Systems

A: Types of Intersections

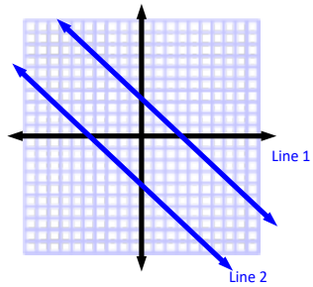
How many solutions can a system of linear equations have?
How can you tell just by looking at the equations? Explain your answer.

A linear system can have:

ONE SOLUTION The lines intersect at one point
The two linear equations have:
- different slopes
- y-intercept does not matter

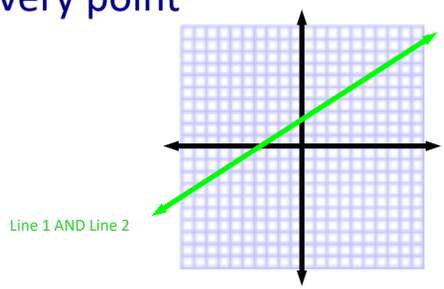


NO SOLUTION The lines do not intersect
The two linear equations have:
- same slopes
- different y-intercepts

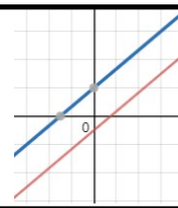
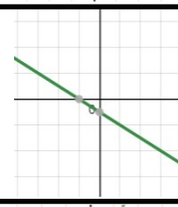
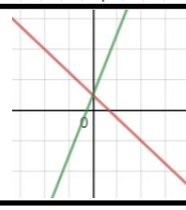


INFINITE SOLUTIONS The lines intersect at every point

The two linear equations have:
- same slopes
- same y-intercepts



Ex. 1 Complete the table.

System #	Equations	Slope	y-int	# of intersections	Solution
1	$y = \frac{2}{3}x - 1$	$\frac{2}{3}$	-1	0	
	$y = \frac{2}{3}x + 2$	$\frac{2}{3}$	2		
2	$y = -\frac{1}{2}x - 1$	$-\frac{1}{2}$	-1	∞	
	$x + 2y = -2$	$-\frac{1}{2}$	-1		
3	$y = -\frac{3}{4}x + 1$	$-\frac{3}{4}$		1	
	$y = 2x + 1$	2			

What kind of equations give an INFINITE number of solutions?

-same slope and same y-intercept

Ex. 2 For what value of p will the system of linear equations have NO solution? – Parallel (same m) – Diff y-ints

a) $x-2y=4$ ①
$y=px+1$ ②

① $x-2y=4$
$-2y=-x+4$
$y=\frac{-1}{-2}x+\frac{4}{-2}$
$=\frac{1}{2}x-2$

Since they need to be parallel (same m)

$\therefore p=\frac{1}{2}$

b) $3x+y=1$ ①
$x+y=p(x+2)$ ②

① $3x+y=1$
$y=-3x+1$
slope $=-3$ y-int $=1$

② $x+y=p(x+2)$
$x+y=px+2p$
$y=px-x+2p$
$=x(p-1)+2p$
$=(p-1)x+2p$
m

$\therefore p-1=-3$
$p=-2$

$\therefore p=-2$

B: Equivalent Systems...

Yesterday, I needed 3 Timmy's coffees and 2 donuts to get the day started. That cost me $8.75. Today, I only needed one coffee and two donuts which cost $4.25. What is the cost of a coffee and a donut?

Let c be the cost of a coffee

Let d be the cost of a donut

① $3c + 2d = 8.75$

② $c + 2d = 4.25$

From ② $c + 2d = 4.25$

$c = 4.25 - 2d$ ③

Sub ③ into ①

$3(4.25 - 2d) + 2d = 8.75$

$12.75 - 6d + 2d = 8.75$

$-4d = -4$

$d = 1$

Sub $d = 1$ into ③

$c = 4.25 - 2(1)$

$= 2.25$

∴ A coffee is $2.25

A donut is $1

Two Tim's lattes and two cookies cost $7.10. Five lattes and four cookies cost $16.95. Determine the cost of a latte and a cookie.

Let l be the cost of a latte

Let c be the cost of a cookie

① $2l + 2c = 7.10$

② $5l + 4c = 16.95$

$l = 2.75$

$c = 0.80$

Homework:

Set 1: p. 32 #1-3, 6

Set 2: p.32 #2, 3d, 8