

2.5 Solving by Elimination

➔ Eliminate means to remove or get rid of.
 What do you think we would like to eliminate?

Consider the following system:

$$\begin{array}{r}
 x - y = 1 \\
 + \quad 3x + y = 11 \\
 \hline
 4x + \cancel{0} = 12
 \end{array}$$

What happens when we add the equations?

We eliminated the y term

Why?

We deliberately set it up to cancel the y-term



To add or to subtract?

Think...

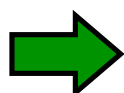
$$\begin{array}{r}
 \boxed{-} \quad y \\
 \quad y \\
 \hline
 0y
 \end{array}$$

$$\begin{array}{r}
 \boxed{+} \quad -y \\
 \quad y \\
 \hline
 0y
 \end{array}$$

$$\begin{array}{r}
 \boxed{+} \quad y \\
 \quad -y \\
 \hline
 0y
 \end{array}$$

$$\begin{array}{r}
 \boxed{-} \quad -y \\
 \quad -y \\
 \hline
 0y
 \end{array}$$

$$\begin{aligned}
 & -y - (-y) \\
 & = -y + y \\
 & = 0
 \end{aligned}$$



Opposite signs --> add
 Same signs --> subtract



Examples: Solve by elimination.

$$\begin{array}{r} \text{a. } x - y = 1 \\ + \quad 3x + y = 11 \\ \hline \end{array}$$

$$4x + 0 = 12$$

$$4x = 12$$

$$x = \frac{12}{4}$$

$$= 3$$

Sub $x=3$ into either eqⁿ
(so pick the easiest!)

Sub $x=3$ into $x-y=1$

$$3 - y = 1$$

$$-y = -2$$

$$y = 2$$

$\therefore (3, 2)$

is the solution

METHOD 2 - THE ELIMINATION METHOD

1. Multiply one or both equations by a constant so that the coefficients of either x or y are the same in both equations (sign does not matter).
2. Add or subtract the equations to *eliminate* one variable.
3. Solve the remaining equation.
4. Substitute the solved value into one of the original equations to determine the value of the other variable.
5. Write a conclusion.
6. Check (formally if asked, otherwise mentally).

b. $x + 3y = 2$ ①

$2x + 5y = 3$ ②

① $\times 2$ $2x + 6y = 4$

② $= 2x + 5y = 3$

 $0 + y = 1$

$y = 1$

Sub $y = 1$ into ①

$x + 3(1) = 2$

$x = 2 - 3$

$x = -1$

$\therefore (-1, 1)$

c. $5x - 3y = 9$ ①

$2x - 5y = -4$ ②

① $\times 2$ $10x - 6y = 18$

② $\times 5$ $- 10x - 25y = -20$

 $0 + 19y = 38$

$y = \frac{38}{19}$

$= 2$

Sub $y = 2$ into into ②

$2x - 5(2) = -4$

$2x - 10 = -4$

$2x = 6$

$x = 3$

$\therefore (3, 2)$

is the solution

d. $2x + 3y = 8$ ①

$3x - 5y = 2$ ②

$$\begin{array}{r} \textcircled{1} \times 3 \quad 6x + 9y = 24 \\ \textcircled{2} \times 2 \quad -6x - 10y = 4 \\ \hline 0 + 19y = 20 \\ y = \frac{20}{19} \end{array}$$

Sub $y = \frac{20}{19}$ into ①

$2x + 3\left(\frac{20}{19}\right) = 8$

$2x + \frac{60}{19} = 8$

$2x = 8 - \frac{60}{19}$

$2x = \frac{152}{19} - \frac{60}{19}$

$2x = \frac{92}{19}$

$$\begin{aligned} x &= \frac{1}{2} \cdot \frac{92}{19} \\ &= \frac{92}{38} \end{aligned}$$

$\therefore \left(\frac{92}{38}, \frac{20}{19}\right)$

e. $3m = -1 - 4n$

$5n = 4m + 22$

Rearrange first!

$3m + 4n = -1$ ①

$-4m + 5n = 22$ ②

$\textcircled{1} \times 4 \quad 12m + 16n = -4$

$\textcircled{2} \times 3 \quad + -12m + 15n = 66$

$0 + 31n = 62$

$n = 2$

Sub $n = 2$ into

$3m + 4(2) = -1$

$3m + 8 = -1$

$3m = -9$

$m = -3$

$\therefore m = -3$

$n = 2$

~~$(m, n) = (-3, 2)$~~



The Eliminator

... solving systems by eliminating one variable at a time!

Homework:

Set 1: p.40 #2d, 4a, 5ab, 6ad, 7c

Set 2: p.40 #5d, 6b, 7c, 12c, 13, 18, 19b, 20a