2.5 Solving by Elimination

Eliminate means to remove or get rid of.
What do you think we would like to eliminate?

Consider the following system:

$$\begin{array}{rl} & x - y = 1 \\ + & 3x + y = 11 \\ \hline & 4x + 0 = 12 \end{array}$$

What happens when we add the equations?

We eliminated the y term

Why?

We deliberately set it up to cancel the y-term

To add or to subtract?
Think...

$$\begin{array}{rl} - & y \\ & y \\ \hline & 0y \end{array} \qquad \begin{array}{rl} + & -y \\ & y \\ \hline & 0y \end{array} \qquad \begin{array}{rl} + & y \\ & -y \\ \hline & 0y \end{array} \qquad \begin{array}{rl} - & -y \\ & -y \\ \hline & 0y \end{array}$$

$$-y - (-y) = -y + y = 0$$

Opposite signs --> add
Same signs --> subtract

Examples: Solve by elimination.

a. $x - y = 1$

$+ \quad 3x + y = 11$

$4x + 0 = 12$

$4x = 12$

$x = \frac{12}{4}$

$= 3$

Sub $x = 3$ into either eqn (so pick the easiest!)

Sub $x = 3$ into $x - y = 1$

$3 - y = 1$

$-y = -2$

$y = 2$

$\therefore (3, 2)$ is the solution

METHOD 2 - THE ELIMINATION METHOD

1. Multiply one or both equations by a constant so that the coefficients of either *x* or *y* are the same in both equations (sign does not matter).
2. Add or subtract the equations to *eliminate* one variable.
3. Solve the remaining equation.
4. Substitute the solved value into one of the original equations to determine the value of the other variable.
5. Write a conclusion.
6. Check (formally if asked, otherwise mentally.

b. $x + 3y = 2$ ①
$2x + 5y = 3$ ②

① × 2 $2x + 6y = 4$
② $-\ 2x + 5y = 3$
$0 + y = 1$
$y = 1$

Sub $y = 1$ into ①
$x + 3(1) = 2$
$x = 2 - 3$
$x = -1$
$\therefore (-1, 1)$

c. $5x - 3y = 9$ ①
$2x - 5y = -4$ ②

① × 2 $10x - 6y = 18$
② × 5 $-\ 10x - 25y = -20$
$0 + 19y = 38$
$y = \frac{38}{19}$
$= 2$

Sub $y = 2$ into into ②
$2x - 5(2) = -4$
$2x - 10 = -4$
$2x = 6$
$x = 3$

$\therefore (3, 2)$
is the solution

d. $2x + 3y = 8$ ①
$3x - 5y = 2$ ②

① × 3 $6x + 9y = 24$
② × 2 $-6x - 10y = 4$
$0 + 19y = 20$
$y = \frac{20}{19}$

Sub $y = \frac{20}{19}$ into ①
$2x + 3\left(\frac{20}{19}\right) = 8$
$2x + \frac{60}{19} = 8$
$2x = 8 - \frac{60}{19}$
$2x = \frac{152}{19} - \frac{60}{19}$
$2x = \frac{92}{19}$
$x = \frac{1}{2} \cdot \frac{92}{19}$
$= \frac{92}{38}$

$\therefore \left(\frac{92}{38}, \frac{20}{19}\right)$

e. $3m = -1 - 4n$
$5n = 4m + 22$

Rearrange first!
$3m + 4n = -1$ ①
$-4m + 5n = 22$ ②

① × 4 $12m + 16n = -4$
② × 3 $+ -12m + 15n = 66$
$0 + 31n = 62$
$n = 2$

Sub $n = 2$ into
$3m + 4(2) = -1$
$3m + 8 = -1$
$3m = -9$
$m = -3$

$\therefore m = -3$
$n = 2$

$(m, n) \Rightarrow (-3, 2)$

The Eliminator

... solving systems by eliminating one variable at a time!

Homework:

Set 1: p.40 #2d, 4a, 5ab, 6ad, 7c
Set 2: p.40 #5d, 6b, 7c, 12c, 13, 18, 19b, 20a