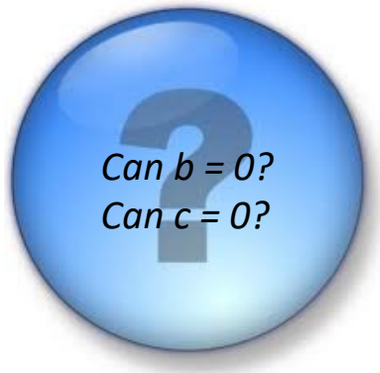
3.2 Quadratic Relations

Standard form

A **quadratic function**'s equation can be written in the form $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$.

Why can't $a = 0$?

$$y = ax^2 + bx + c$$
$$y = 0\left(x^2\right) + bx + c$$
$$y = bx + c$$ ← This is linear!



Here are three examples of quadratic relations; state the values of a, b and c.

$y = 2x^2 + 3x + 1$ $a = 2$ $b = 3$ $c = 1$

$y = 5x^2 - 4$ $a = 5$ $b = 0$ $c = -4$

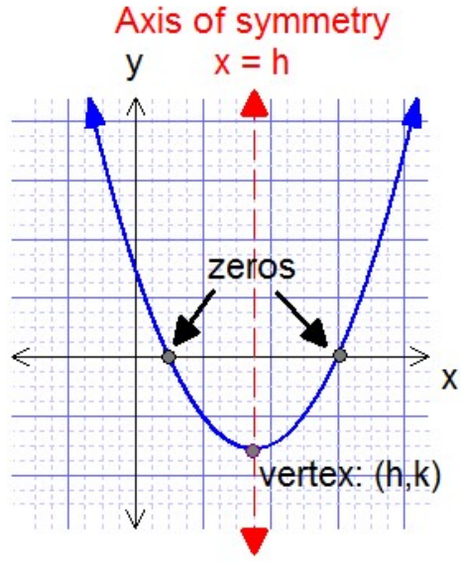
$y = x^2$ $a = 1$ $b = 0$ $c = 0$

Features of Quadratics

- The ***vertex*** of a parabola is either the minimum point (opens up) or maximum point (opens down).

- A vertical line of symmetry which goes through the vertex is called the ***axis of symmetry***.

- The x-intercept(s) of a parabola are called its ***zeros*** or roots.



How can you tell if data is linear?

the first differences are constant

Let's examine some data from the last lesson. What patterns do you notice?

x	y	1ST Differences	2ND Differences
-3	23		
		-13	
-2	10		+4
		-9	
-1	1		+4
		-5	
0	-4		+4
		-1	
1	-5		+4
		+3	
2	-2		+4
		+7	
3	5		

2ND Differences are CONSTANT!

∴ Quadratic

- Linear Relation: if a relation has constant first differences (ie. slope) the relation is linear.
- Quadratic Relation: if a relation has constant second differences the relation is quadratic.

both depend on a constant increment of the independent variable

Ex.3 Calculate the first and second differences to determine whether the relation is linear, quadratic, or neither.

x	y
-1	5
0	7
1	9
2	11
3	13

7 - 5 = 2, 2, 2

∴ LINEAR!
1ST Diff are constant

x	y
-2	3
-1	-3
0	-5
1	-3
2	3

-3 - 3 = -6, -5 - (-3) = -2, 2, 6; +4, +4, +4

∴ Quadratic
2ND Diff are constant

x	y
-3	7
0	4
3	1
6	-2
9	-5

-3, -3, -3, -3

∴ LINEAR

x	y
1	4
2	6
3	12
4	18
5	28

+2, +6, +6, +10; +4, +0, +4

∴ Neither!

Neither 1ST or 2ND differences are constant

Applications

Ex. 4 The path of a golf ball is modelled by the equation $y = -x^2 + 5x$, where x represents the horizontal distance travelled by the ball in metres and y represents the height of the ball in metres.

a) Complete the table of values and graph the relation.

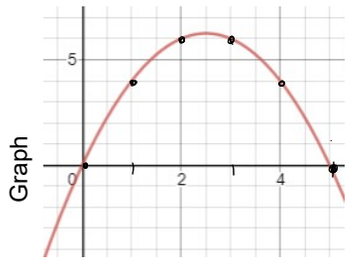
$y = -x^2 + 5x$

$y = -(0)^2 + 5(0)$
$= 0$

$y = -(1)^2 + 5(1)$
$= -1 + 5$
$= 4$

$y = -(2)^2 + 5(2)$
$= -4 + 10$
$= 6$

x	y
0	0
1	4
2	6
3	6
4	4
5	0
6	-6



b) Determine the coordinates of the vertex.

Axis of symmetry: $\frac{0+5}{2} = 2.5$

Sub it in to find y-value!

$y = -(2.5)^2 + 5(2.5)$
$= 6.25$

$\therefore (2.5, 6.25)$

c) What was the maximum height of the ball?

Max height is 6.25m

d) How far away does the ball land?

The ball lands at 5m (height is 0 at $x=5$)

e) What was the height of the ball 4 m away from the golfer?

The ball was 4m high at $x = 4$

Homework

SET 1: P. 172 #1 (GRAPH BY HAND)
#2, 6, 9 (WITH DESMOS)

SET 1: P. 172 #1 (GRAPH BY HAND)
#2, 5, 6, 9 (WITH TECHNOLOGY)