2.2A Operations with Rational Expressions (Multiplying and Dividing)

A concrete example:

$$\frac{2}{\cancel{9}_{3}}\times\frac{\cancel{\cancel{6}}^{\,3\ 1}}{\cancel{14}_{7}}$$

$$=\frac{2}{21}$$

Simplify before multiplying.

$$\frac{\cancel{2}^{1}}{{}_{3}\cancel{9}}\cdot\frac{\cancel{6}^{2}}{\cancel{14}_{7}}$$

A concrete example:

$$\frac{5}{6}\div\frac{10}{9}$$

$$=\frac{\cancel{5}^{1}}{\cancel{6}_{2}}\times\frac{\cancel{9}^{3}}{\cancel{10}_{2}}$$

$$=\frac{3}{4}$$

Multiply by the reciprocal of the divisor, then simplify.

Ex.1 Simplify the rational expression. State the restrictions.

$$\frac{15ab^3}{4a}\div\frac{25a^4b}{12ab^4}$$

$$=\frac{\cancel{15}^{3}\cancel{a}b^3}{{}_{1}\cancel{4}\cancel{a}}\times\frac{{}^{3}\cancel{12}\cancel{a}b^{\cancel{4}\,3}}{{}_{5}\cancel{25}a^{\cancel{4}}_{3}\cancel{b}}$$

$$=\frac{9b^6}{5a^3},\quad a\neq 0,\ b\neq 0$$

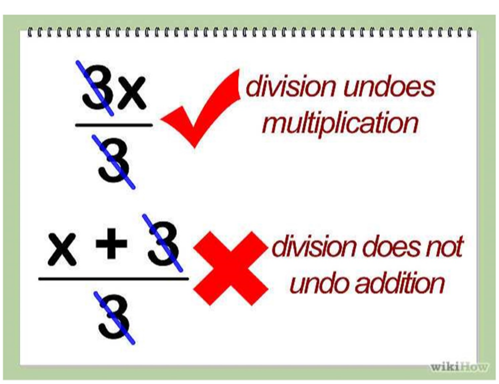
This is an easier example because the expressions are monomials.

Ex.2 Simplify the rational expression. State the restrictions.

$$\frac{\cancel{x+1}}{x-1}\bullet\frac{x-2}{\cancel{x+1}}$$

$$=\frac{x-2}{x-1},\quad x\neq\pm 1$$

This is an easier example because the expressions are factored.



Divide out factors... not terms!!

Ex.3 Simplify the rational expressions. State restrictions.

PROCESS
1. Factor both the numerator and denominator.
2. IF dividing, multiply by the reciprocal of the divisor.
3. Simplify/reduce by any factors common to any numerator and denominator (diagonal and top/bottom).
4. State restrictions.

a) $\dfrac{4x-6}{8x^2y} \times \dfrac{4xy}{6x-9}$

$$= \frac{\cancel{2}(\cancel{2x-3})}{\cancel{8}x^{\cancel{2}}\cancel{y}} \cdot \frac{\cancel{4}\cancel{x}\cancel{y}}{3(\cancel{2x-3})}$$

$$= \frac{1}{3x}, \quad x \neq 0, \frac{3}{2}, \quad y \neq 0$$

Restrictions are stated for ~~all values~~ of the variables that WERE and ARE in the denominator.

EVER, at all, at any point

b) $\dfrac{x^2+4x+4}{x-2} \div \dfrac{3x+6}{x^2-5x+6}$

$$= \frac{(x+2)(x+2)}{x-2} \div \frac{3(x+2)}{(x-2)(x-3)}$$

$$= \frac{(x+2)\cancel{(x+2)}}{\cancel{x-2}} \times \frac{\cancel{(x-2)}(x-3)}{3\cancel{(x+2)}}$$

$$= \frac{(x+2)(x-3)}{3}, \quad x \neq \pm 2, 3$$

Ex.4 Simplify and state restrictions.

$$\frac{2x^2-5x-3}{2x^2-11x+15}\times\frac{4x^2-8x-5}{4x^2+4x+1}$$

M -6
A -5
N $\frac{-6}{2}$, $\frac{1}{2}$
$-\frac{3}{1}$, $\frac{1}{2}$

M -20
A -8
N $\frac{-10}{4}$, $\frac{2}{4}$
$-\frac{5}{2}$, $\frac{1}{2}$

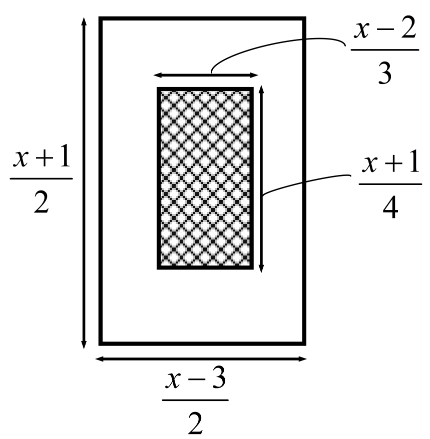
$$= \frac{\cancel{(x-3)}\cancel{(2x+1)}}{\cancel{(x-3)}\cancel{(2x-5)}}\times\frac{\cancel{(2x-5)}\cancel{(2x+1)}}{\cancel{(2x+1)}\cancel{(2x+1)}}$$

$$= 1, \quad x \neq 3, -\frac{1}{2}, \frac{5}{2}$$

M 30
A -11
N $\frac{-6}{2}$, $\frac{-5}{2}$
$-\frac{3}{1}$, $\frac{-5}{2}$

M 4
A 4
N $\frac{2}{4}$, $\frac{2}{4}$
$\frac{1}{2}$, $\frac{1}{2}$

Ex.5 a) Write and simplify an expression that represents the ratio of the large rectangular area to the shaded rectangular area.
b) What are the restrictions on x? What are the restrictions in the context of the problem?



$$\frac{A_{LARGE}}{A_{SHADED}}$$

$$A_{LARGE} = \frac{x-3}{2}\cdot\frac{x+1}{2} = \frac{(x-3)(x+1)}{4}$$

$$A_{SHADED} = \frac{x-2}{3}\cdot\frac{x+1}{4} = \frac{(x-2)(x+1)}{12}$$

$$\frac{A_{LARGE}}{A_{SHADED}}$$

$$= \frac{(x-3)(x+1)}{4} \div \frac{(x-2)(x+1)}{12}$$

$$= \frac{(x-3)\cancel{(x+1)}}{{}_1\cancel{4}}\times\frac{{}^3\cancel{12}}{(x-2)\cancel{(x+1)}}$$

$$= \frac{3(x-3)}{x-2}, \quad x \neq 2, -1$$

HOMEWORK

Pg. 121 # 1bc, 2b, 3, 5cd, 7ac

Additional HW Handout Lesson 2.2A