Lesson 2.3B: Horizontal and Vertical Translations of Functions

Part A: Vertical Translations

Using Desmos, describe the transformations to the base graph in each case.

Graph a couple of equations at a time so that you can see the transformation from the base function.

a) $f(x) = x^2$ BASE FUNCTION
b) $g(x) = f(x) + 5$ graph moves SHIFT UP 5
c) $h(x) = f(x) - 3$ graph moves DOWN 3

d) $f(x) = \sqrt{x}$ BASE FUNCTION
e) $g(x) = f(x) + 4$ graph moves UP 4
f) $h(x) = f(x) - 2$ graph moves DOWN 2

Think about the Domain and Range as you graph....

Try graphing the base function along with each of these:

g) $m(x) = \frac{1}{x} + 3$ Base Function: $\frac{1}{x}$ graph moves UP 3

h) $n(x) = x^3 - 5$ Base Function: x^3 graph moves DOWN 5

General Result

g(x) = f(x) + c is a vertical translation of the graph of f(x).

If c > 0, the graph of *f(x)* moves UP c units
If c < 0, the graph of *f(x)* moves DOWN c units

The domain does NOT change The range CAN change.
x-values are unaffected. y-values are affected.

c is OUTSIDE of the function so no x-values change.

Part B - Horizontal Translations

Graph the following using Desmos and compare to the base function.

1. Graph $f(x) = x^2$ and the equations below. Describe the transformations.

   a) $g(x) = f(x+4)$ LEFT 4

   b) $h(x) = f(x-2)$ RIGHT 2

2. Graph $f(x) = \sqrt{x}$ and the equations below. Describe the transformations.

   a) $g(x) = f(x+1)$ LEFT 1

   b) $h(x) = f(x-4)$ RIGHT 4

General Result

g(x) = f(x - d) is horizontal translation of the graph of f(x).

If d > 0, the graph of *f(x)* moves RIGHT d units

If d < 0, the graph of *f(x)* moves LEFT d units

The domain can change. The range does NOT change

x-values are affected. y-values are not affected.

d is INSIDE the function so no y-values change.

This transformation is the opposite of what you think because the x-coord must compensate for its change in order for the y-coord to stay the same.

Ex. 1: Given the graph of $f(x)$ shown below, graph:

Graphing Process

- Plot 3 to 5 base points from the parent function.
- Transform these points in order to create the graph.

OR

- Use mapping notation to find the coordinates of the transformed points.

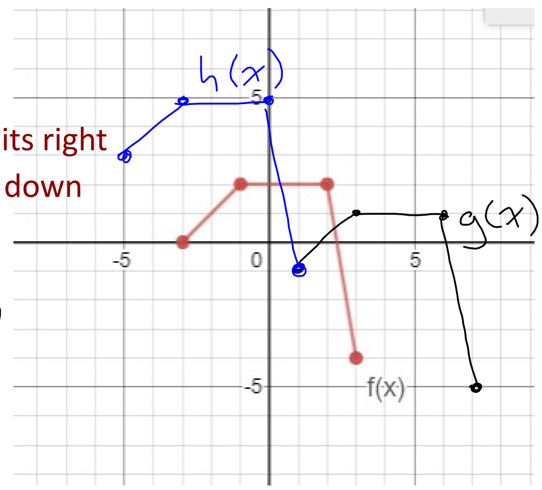
a) $g(x) = f(x-4)-1$

- horizontal translation 4 units right
- vertical translation 1 units down

Solution

$(x+4, y-1)$

$(3,-4) \rightarrow (7,-5)$



b) $h(x) = f(x+2)+3$

- h.t. 2 units left
- v.t. 3 units up

Mapping Notation

$(x,y) \rightarrow (x+d, y+c)$

Solution

Ex. 2: Find the equation of $g(x) = f(x+1)-3$ if:

a) $f(x) = x^2$

$g(x) = f(x+1) - 3$
$= (x+1)^2 - 3$

b) $f(x) = x^3$

$g(x) = f(x+1) - 3$
$= (x+1)^3 - 3$

c) $f(x) = \sqrt{x}$

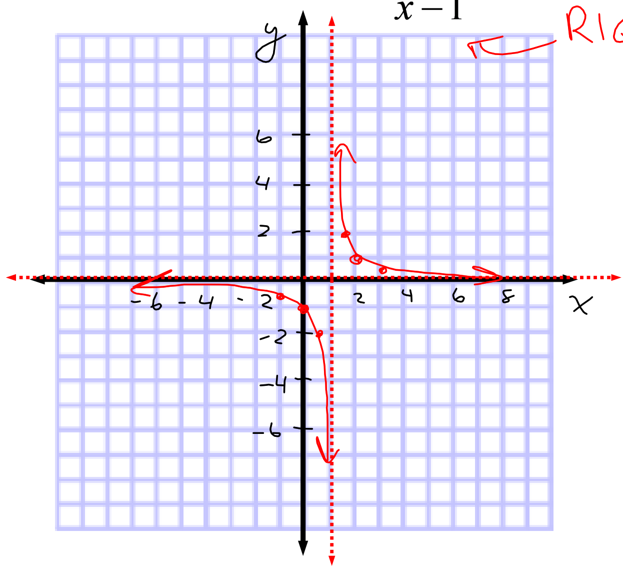
$g(x) = f(x+1) - 3$
$= \sqrt{x+1} - 3$

d) $f(x) = \dfrac{1}{x}$

$g(x) = f(x+1) - 3$
$= \dfrac{1}{x+1} - 3$

Ex. 3: a) Graph $f(x)=\dfrac{1}{x-1}$.

RIGHT 1



b) State the domain and range.

$D:\{x\in R \mid x\neq \cancel{0}\ 1\}$

$R:\{y\in R \mid y\neq 0\}$ ✓

$(x+1,\ y)$

$\left(\tfrac{1}{2},2\right)\rightarrow\left(\tfrac{3}{2},2\right)$

$(1,1)\rightarrow(2,1)$

$\left(2,\tfrac{1}{2}\right)\rightarrow\left(3,\tfrac{1}{2}\right)$

$(-1,-1)\rightarrow(0,-1)$

$\left(-\tfrac{1}{2},-2\right)\rightarrow\left(\tfrac{1}{2},-2\right)$

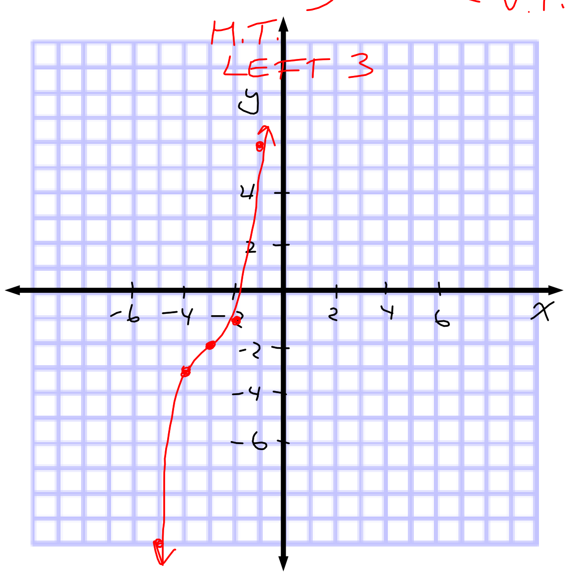
$\left(-2,-\tfrac{1}{2}\right)\rightarrow\left(-1,-\tfrac{1}{2}\right)$

Solution

b) Graph $f(x)=(x+3)^3-2$.

H.T. LEFT 3

V.T. DOWN 2



b) State the domain and range.

$D:\{x\in R\}$ ✓

$R:\{y\in R\}$ ✓

$(x,y)\rightarrow(x-3,\ y-2)$

$(-2,-8)\rightarrow(-5,-10)$

$(-1,-1)\rightarrow(-4,-3)$

$(0,0)\rightarrow(-3,-2)$

$(1,1)\rightarrow(-2,-1)$

$(2,8)\rightarrow(-1,6)$

Solution

HOMEWORK

p. 51 #1

p. 70 # 7a, 8a, 10ab

+ Extra Practice Sheet 2.3B