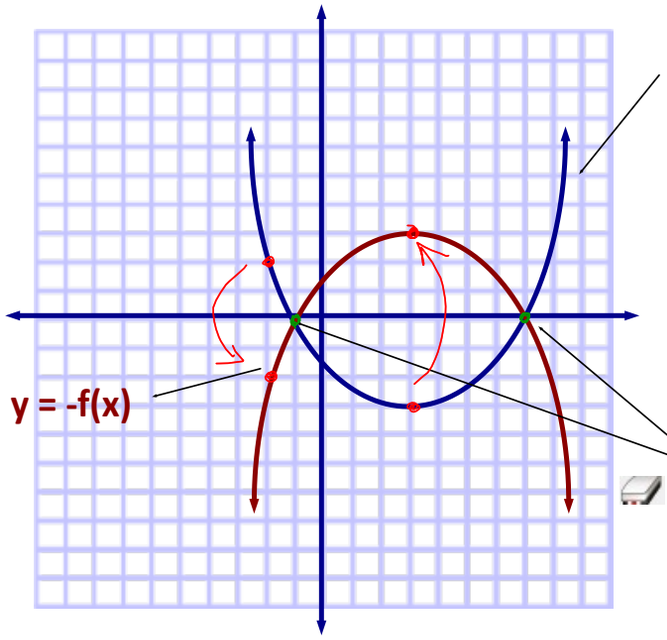
Lesson 2.4: Reflections

A reflection creates a mirror image.

A. Reflection in the x-axis (vertical reflection).



What do you notice about the y-values?

y-values are flipped

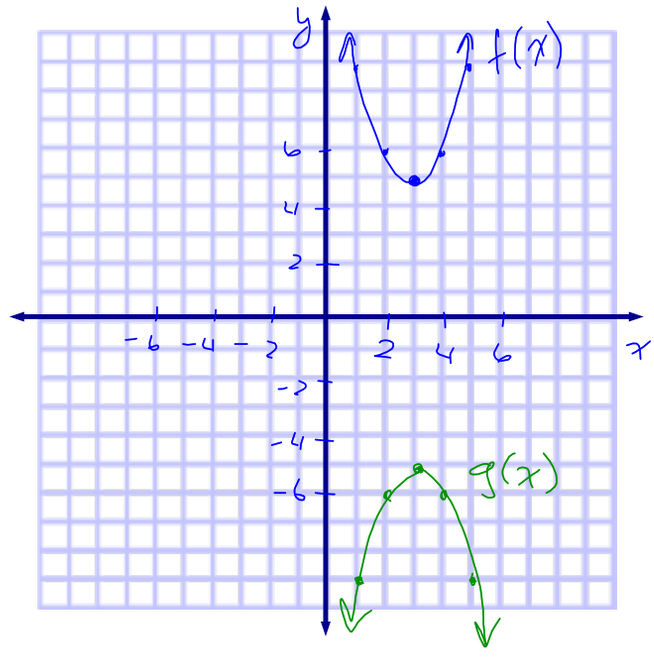
What do you notice about the x-values?

no change

Invariant points are on the x-axis.

Ex. 1:

a) Graph $f(x) = (x - 3)^2 + 5$



b) Find the equation of g(x) = -f(x).

$g(x) = -\left[(x-3)^2 + 5\right]$

$= -(x-3)^2 - 5$

c) Graph g(x).

d) List any invariant points.

e) State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} \mid y \geq 5\}$

g(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} \mid y \leq -5\}$

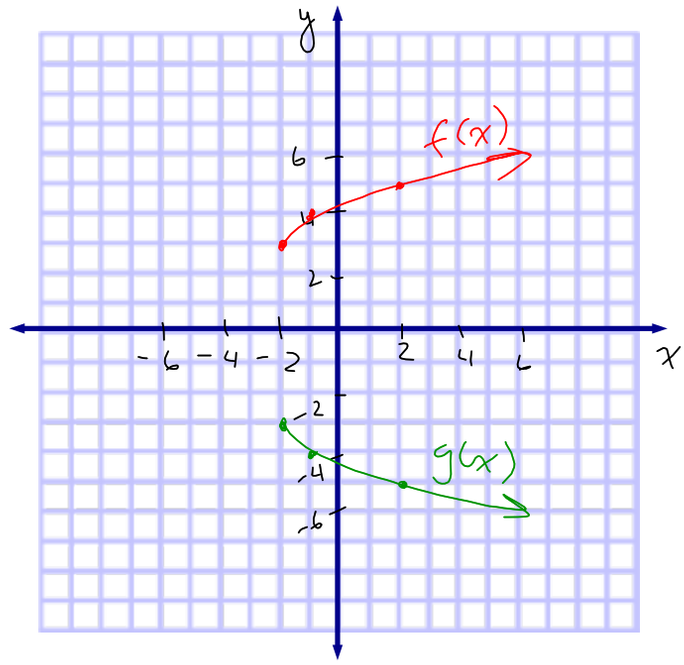
Ex. 2: $(x-2, y+3)$

a) Graph f(x) = $\sqrt{x+2}+3$

$(0,0) \to (-2,3)$
$(1,1) \to (-1,4)$
$(4,2) \to (2,5)$

b) Find the equation of g(x) = -f(x).

$g(x) = -[\sqrt{x+2} + 3]$
$= -\sqrt{x+2} - 3$



c) Graph g(x).

d) List any invariant points.

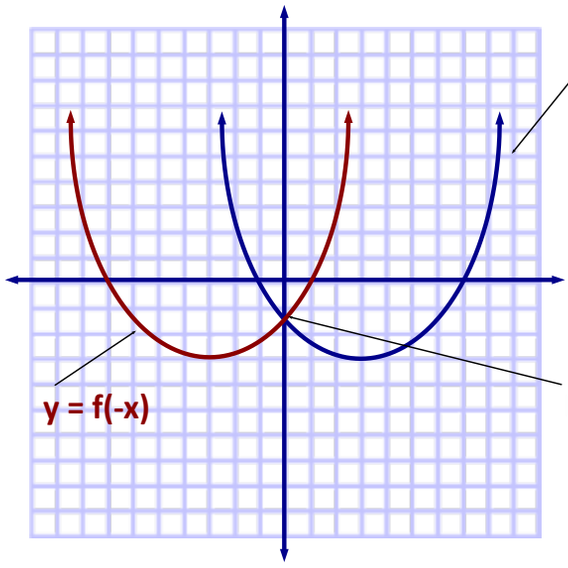
None

e) State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R} \mid x \geq -2\}$
R= $\{y \in \mathbb{R} \mid y \geq 3\}$

g(x): D= $\{x \in \mathbb{R} \mid x \geq -2\}$
R= $\{y \in \mathbb{R} \mid y \leq -3\}$

B. Reflection in the y-axis (horizontal reflection).



What do you notice about the x-values?

changed (flipped)

What do you notice about the y-values?

did not change

Invariant points are on the y-axis.

Ex. 3:

a) Graph $f(x) = (x + 4)^2$

Replace "x" with "-x".

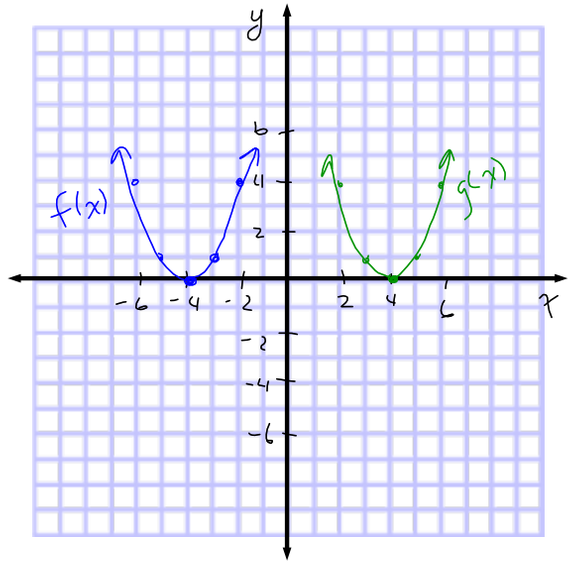
b) Find the equation of g(x) = f(-x).

$g(x) = (-x + 4)^2$

$= [(-1)(x - 4)]^2$

$= (-1)^2 (x - 4)^2$

$= (x - 4)^2$



c) Graph g(x).

d) List any invariant points.

y-intercept! Sub $x = 0$

$f(0) = (0 + 4)^2$

$= 16$

$\therefore (0, 16)$

e) State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R}\}$

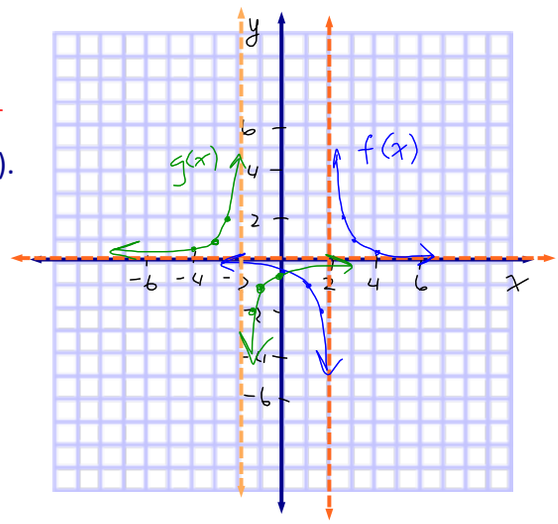
R= $\{y \in \mathbb{R} \mid y \geq 0\}$

g(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} \mid y \geq 0\}$

Ex. 4

a) Graph f(x) = $\frac{1}{x-2}$ ← RIGHT 2



b) Find the equation of g(x) = f(-x).

$g(x) = \frac{1}{-x-2}$

$= \frac{1}{-(x+2)}$

Vert. Refl. $= -\frac{1}{x+2}$ ← LEFT 2

c) Graph g(x).

d) List any invariant points.

y-ints: $f(0) = \frac{1}{0-2} = -\frac{1}{2}$ $\therefore (0, -\frac{1}{2})$

e) State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R} \mid x \neq 2\}$ R= $\{y \in \mathbb{R} \mid y \neq 0\}$

g(x): D= $\{x \in \mathbb{R} \mid x \neq -2\}$ R= $\{y \in \mathbb{R} \mid y \neq 0\}$

Summary

If y = f(x), then:

y = - f(x) represents a reflection in x-axis.
Under a reflection in the x-axis, the point (x, y) ⇒(x, -y).
Invariant points are located on the x-axis (x-ints).

y = f(-x) represents a reflection in y-axis.
Under a reflection in the y-axis, the point (x, y) ⇒(-x, y).
Invariant points are located on the y-axis (y-ints).

y = - f(-x) represents a reflection in x-axis and y-axis. The order of these reflections does not matter. Under these reflections, the point (x, y) ⇒ (-x, -y).

Homework

Handout Sheet 2.4

Earlier Practice Sheet 2.4