

**Lesson 2.5: Stretches/Compressions of Functions** Desmos

**Part A: Vertical Stretches & Compressions**  $g(x) = af(x)$

$g(x) = af(x)$  is the graph of  $f(x)$  that has been vertically stretched by a factor of "a".

If  $a > 1$ , then the graph is vertically stretched.

If  $0 < a < 1$ , then the graph is vertically compressed.

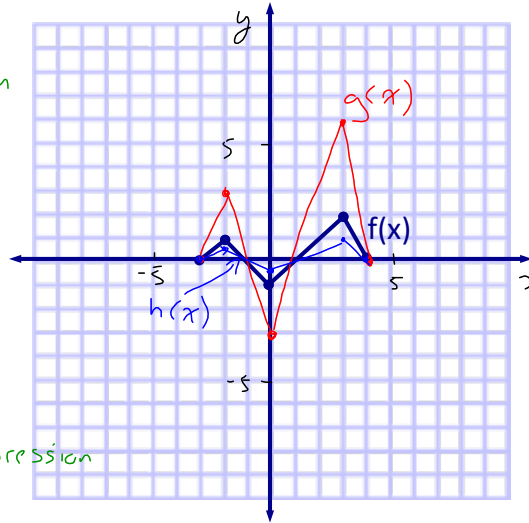
Ex. 1: Given  $f(x)$  as shown, graph:

a)  $g(x) = 3f(x)$  Vertical stretch by 3

- $(x,y) \rightarrow (x, 3y)$
- $(-3, 0) \rightarrow (-3, 0)$
  - $(-2, 1) \rightarrow (-2, 3)$
  - $(0, -1) \rightarrow (0, -3)$
  - $(3, 2) \rightarrow (3, 6)$
  - $(4, 0) \rightarrow (4, 0)$

b)  $h(x) = \frac{1}{2}f(x)$  Vertical compression by 2

$(x,y) \rightarrow (x, \frac{1}{2}y)$



Which points are invariant?  $\Rightarrow$  Points lying on the x-axis (y-coord = 0)

Ex. 2: Given  $f(x) = x^2$  write equations to represent  $g(x)$  and  $h(x)$  and graph :

a)  $g(x) = 2f(x)$  Vertical stretch by 2

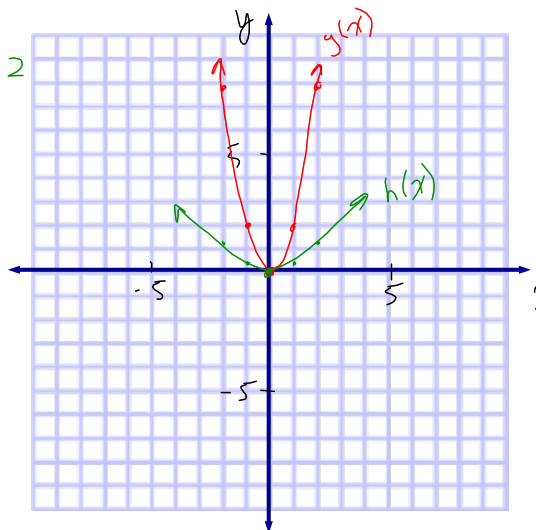
$(x,y) \rightarrow (x, 2y)$

- $(-2, 4) \rightarrow (-2, 8)$
- $(-1, 1) \rightarrow (-1, 2)$
- $(0, 0) \rightarrow (0, 0)$
- $(1, 1) \rightarrow (1, 2)$
- $(2, 4) \rightarrow (2, 8)$

b)  $h(x) = \frac{1}{3}f(x)$  Vertically compressed by 3

$(x,y) \rightarrow (x, \frac{y}{3})$

- $(0, 0) \rightarrow (0, 0)$
- $(1, 1) \rightarrow (1, \frac{1}{3})$
- $(2, 4) \rightarrow (2, \frac{4}{3})$



What do you notice about the domain and range?  $\Rightarrow$  The domain is not affected by a vertical transformation. The range is affected.

**Part B: Horizontal Stretches & Compressions**  $g(x) = f(kx)$

Note: Use Desmos again.

$g(x) = f(kx)$  is the graph of  $f(x)$  that has been horizontally stretched by a factor of " $\frac{1}{k}$ ".

If  $k > 1$ , then the graph is horizontally compressed.

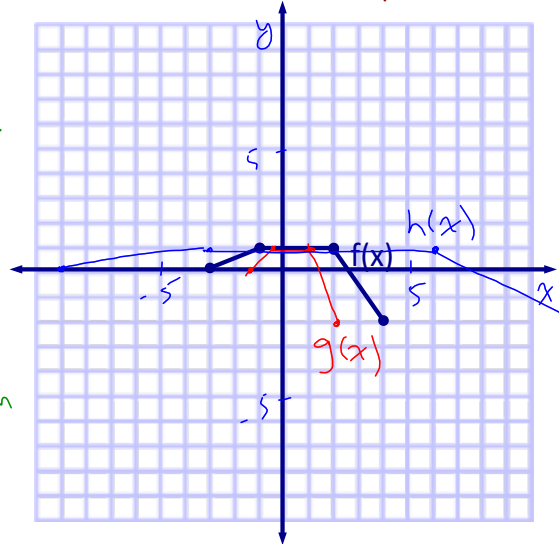
If  $0 < k < 1$ , then the graph is horizontally stretched.

Note:  $k$  does the opposite of what you naturally think since it is inside the function.  
 Note: Textbook uses incorrect terminology for both vertical and horizontal compressions!

Ex. 3: Given  $f(x)$ , graph:

a)  $g(x) = f(2x)$  Horiz. Comp. by 2  
 $(x, y) \rightarrow (\frac{x}{2}, y)$   
 $(-3, 0) \rightarrow (-\frac{3}{2}, 0)$   
 $(4, 0) \rightarrow (2, 0)$

b)  $h(x) = f(\frac{1}{3}x)$  Horiz. Stretch by 3  
 $(x, y) \rightarrow (3x, y)$

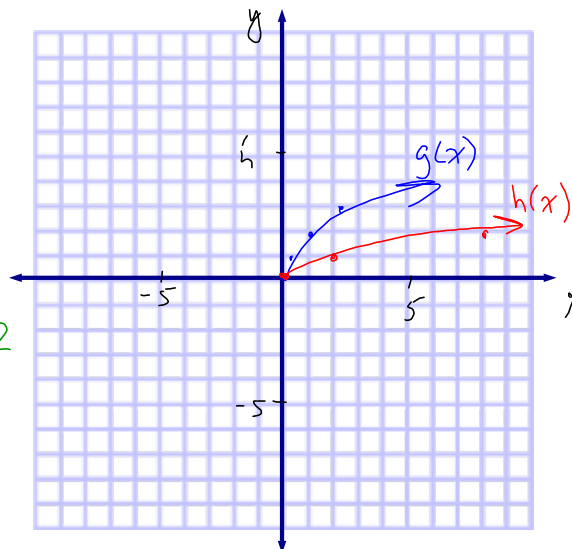


Which points are invariant?  $\Rightarrow$  Points lying on the y-axis (x-coord = 0) 📱

Ex. 4: Given  $f(x) = \sqrt{x}$  write equations to represent  $g(x)$  and  $h(x)$  and graph :

a)  $g(x) = f(4x)$  H.C. by 4  
 $(x, y) \rightarrow (\frac{x}{4}, y)$   
 $(0, 0) \rightarrow (0, 0)$   
 $(1, 1) \rightarrow (\frac{1}{4}, 1)$   
 $(4, 2) \rightarrow (1, 2)$   
 $(9, 3) \rightarrow (\frac{9}{4}, 3)$

b)  $h(x) = f(\frac{1}{2}x)$  H.S. by 2  
 $(x, y) \rightarrow (2x, y)$   
 $(0, 0) \rightarrow (0, 0)$   
 $(1, 1) \rightarrow (2, 1)$   
 $(4, 2) \rightarrow (8, 2)$



What do you notice about the domain and range?  $\Rightarrow$  The range is not affected by a horizontal transformation. The domain is affected. 📱

## Part C: Combining Horizontal &amp; Vertical Stretches &amp; Compressions

Ex. 5: Given  $f(x) = |x|$ :

a) Write an equation to represent

$$g(x) = 2f(2x)$$

$$= 2|2x|$$

①      ②

b) Describe the transformations.

① V.S. by 2

② H.C. by 2

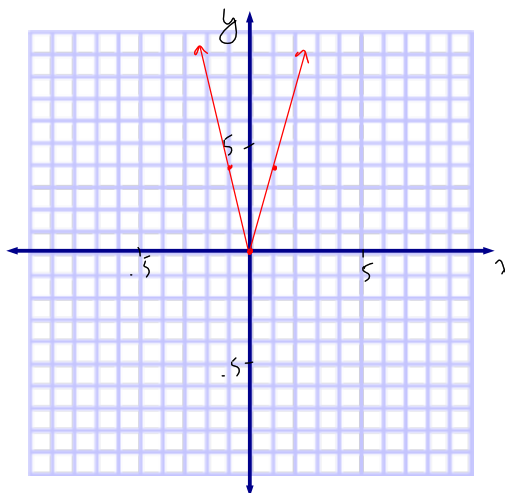
c) Graph  $g(x) = 2f(2x)$ .

$$(x, y) \rightarrow \left(\frac{x}{2}, 2y\right)$$

$$(-2, 2) \rightarrow (-1, 4)$$

$$(0, 0) \rightarrow (0, 0)$$

$$(2, 2) \rightarrow (1, 4)$$



d) State the domain and range.

$$D: \{x \in \mathbb{R}\}$$

$$R: \{y \in \mathbb{R} \mid y \geq 0\}$$

Ex. 6: Given that  $f(x) = (2x)^2$  is a parabola that has been horizontally compressed by a factor of 2, can you describe a different transformation that would give the SAME graph?

$$f(x) = (2x)^2 \leftarrow \text{H.C. by 2}$$

$$= 2^2 x^2$$

$$= 4x^2 \leftarrow \text{V.S. by 4}$$

$$f(x) = \sqrt{x}$$

$$g(x) = f(9x)$$

H.C. by 9

$$= \sqrt{9x}$$

$$= \sqrt{9} \sqrt{x}$$

$$= 3\sqrt{x}$$

$$h(x) = 3f(x)$$

V.S. by 3

$$= 3\sqrt{x}$$

SAME!

## Homework

p. 58 # 3, 4ab, 5ab, 6ab, 7ab

Use the same two graphs!

10b, 12

## Extra Practice 2.5

