# Lesson 2.6A: Combinations of Transformations

## Summary of Transformations

If y = f(x), then describe the effect of a, k, d and c.

d represents horizontal translation
if d < 0 left "d" units
if d > 0 right "d" units

c represents vertical translation
if c < 0 down "c" units
if c > 0 up "c" units

$$y = a\, f[\, k\,(x - d)\,] + c$$

a represents vertical stretch/compres. and/or reflection
if 0 < a < 1 vertical compression by "$\frac{1}{a}$"
if a > 1 vertical stretch by "a"
if a < 0 reflection in the x-axis

k represents horizontal stretch/compr and/or reflection
if 0 < k < 1 horiz. stretch of "$\frac{1}{k}$"
if k > 1 horiz. compression of "k"
if k < 0 reflection in the y-axis

## Order of Transformations

1st: Transformations that are multiplied/divided
- Stretching / Compressing
- Reflecting

2nd: Transformations that are added/subtracted.
- Translations

Ex. 1: Describe the following transformations on a function y = f(x).

| Function | Transformations to apply first | Transformations to apply second |
|---|---|---|
| $y = -2f(x - 3) + 1$<br>ex: $y = -2\lvert x-3\rvert + 1$ | • Reflection in the x-axis<br>• Vertical stretch of 2 | • Horizontal translation to the right 3 units<br>• Vertical translation up 1 unit |
| $y = f(3(x + 2)) - 5$<br>ex: $y = \sqrt{3(x+2)} - 5$ | • Horizontal compression by 3 | • Horizontal translation to the left 2 units<br>• Vertical translation down 5 units |
| $y = -5f(-2(x - 4)) - 1$<br>ex: $y = -5(-2(x-4))^3 - 1$ | • Reflection in the x-axis<br>• Vertical stretch of 5<br>• Reflection in the y-axis<br>• Horizontal compression by 2 | • Horizontal translation to the right 4 units<br>• Vertical translation down 1 unit |

If the transformed equation is not given in standard form, you will have to factor in order to determine the correct order of the transformations.

$y = a\,f[\,k\,(x - d)\,] + c$

Ex. 2: Identify the transformations that have been applied to each of the following base functions.

a) $y = f(3x + 6)$ factor!

$y = f[3(x+2)]$

- Horz. Comp. by 3 "H.C. by 3"
- Horz. Trans. Left 2 "Left 2"

b) $y = \sqrt{-x + 5}$ FACTOR!

$y = \sqrt{-(x-5)}$

- Reflected over y-axis
- Right 5

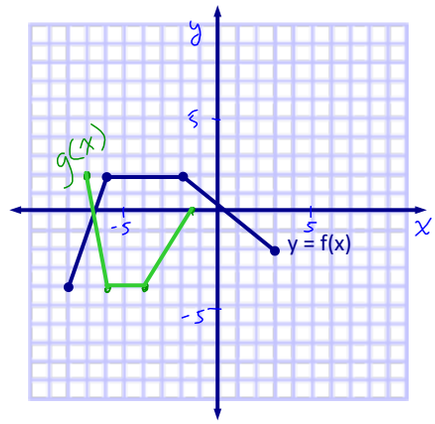
$$y = a\,f[\,k\,(x - d)\,] + c$$

$$(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

Ex. 3: Given the graph of f(x), transform the function as indicated.

g(x) = - f(2x + 6) - 2 FACTOR.

$= -f[2(x+3)] - 2$




$(x, y) \rightarrow \left(\frac{x}{2} - 3, -y - 2\right)$

$(-8, -4) \rightarrow (-7, 2)$

$(-6, 2) \rightarrow (-6, -4)$

$(-2, 2) \rightarrow (-4, -4)$

$(3, -2) \rightarrow \left(-\frac{3}{2}, 0\right)$

Ex. 4: Write the equation of the transformed function using the given base function and transformations.

a) $f(x) = x^2$, reflected in the x-axis, vertical stretch by 3, left 6, down 2.

$g(x) = -3(x+6)^2 - 2$

b) $f(x) = \frac{1}{x}$, horizontal stretch by 2, reflection in the y-axis, right 7, up 3.

$f\left(-\frac{1}{2}(x-7)\right) + 3$

$g(x) = \frac{1}{-\frac{1}{2}(x-7)} + 3$

c) $f(x) = \sqrt{x}$, horizontal compression by 3, reflection in the x-axis, reflection in the y-axis, left 6, down 2.

$y = a\,f[k(x-d)] + c$

$= a\sqrt{k(x-d)} + c$

$= -\sqrt{-3(x+6)} - 2$

Ex. 5: Given f(x), write the corresponding equation and graph the transformed function.

a) $f(x) = x^3$ , graph $g(x) = -2f(3 - x) + 4$

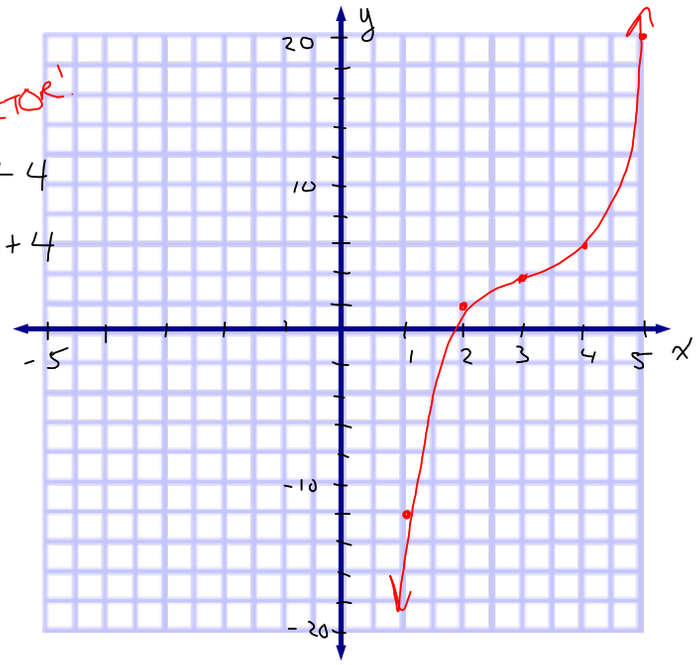
FACTOR!

$= -2f(-x+3)+4$

$= -2f(-(x-3))+4$

$(x,y) \to (-x+3, -2y+4)$

$(-2,-8) \to (5,20)$
$(-1,-1) \to (4,6)$
$(0,0) \to (3,4)$
$(1,1) \to (2,2)$
$(2,8) \to (1,-12)$




b) $f(x) = \sqrt{x}$, graph $g(x) = f(-2x + 6) - 5$

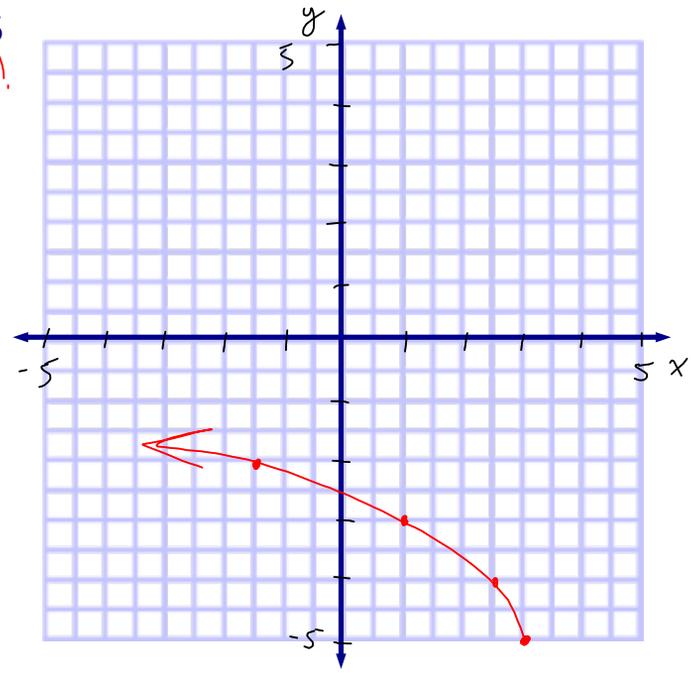
FACTOR!

$g(x) = f[-2(x-3)] - 5$

$(x,y) \to (-\frac{x}{2}+3, y-5)$

Base points for $\sqrt{x}$ ?

$(0,0) \to (3,-5)$
$(1,1) \to (\frac{5}{2},-4)$
$(4,2) \to (1,-3)$
$(9,3) \to (-\frac{3}{2},-2)$



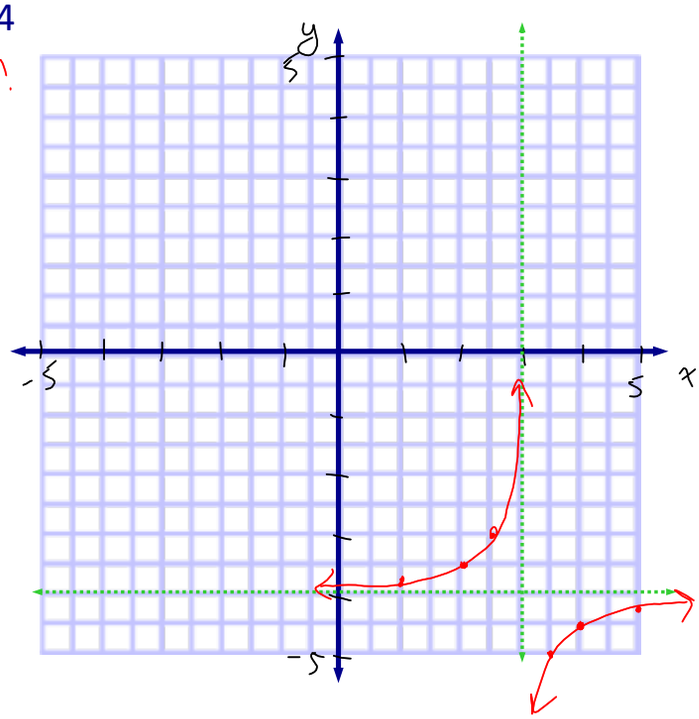
c) $f(x) = \frac{1}{x}$, graph $g(x) = \frac{1}{2}f(-x + 3) - 4$

FACTOR!

$$g(x) = \frac{1}{2} f\left[-(x-3)\right] - 4$$

$$(x, y) \rightarrow \left(-x+3 \, , \, \frac{1}{2}y - 4\right)$$

$\left(-2, -\frac{1}{2}\right) \rightarrow (5, -4.25)$
$(-1, -1) \rightarrow (4, -4.5)$
$\left(-\frac{1}{2}, -2\right) \rightarrow (3.5, -5)$
$\left(\frac{1}{2}, 2\right) \rightarrow (2.5, -3)$
$(1, 1) \rightarrow (2, -3.5)$
$\left(2, \frac{1}{2}\right) \rightarrow (1, -3.75)$



# Homework
# p.70 # 1, 3, 4acd, 7c, 8ac, 10abe, 16, 18, 19a
# Extra Practice 2.6A