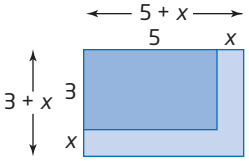
5.1 Multiply Polynomials

A rectangular garden measures 3 m by 5 m. If each dimension is increased by the same amount to expand the garden, how can you model the area of the new garden using a polynomial?



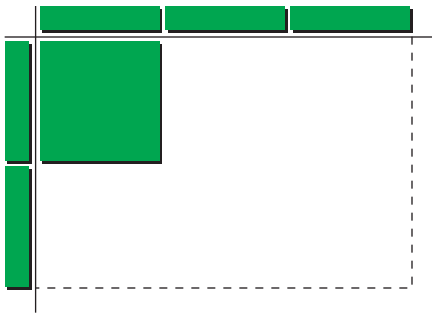
Investigate A

How can you model the multiplication of polynomials?

Tools
- algebra tiles

Method 1: Use Algebra Tiles

1. To show the product $(2x)(3x)$, use algebra tiles to model $2x$ and $3x$ as the dimensions of a rectangle. Then, fill in the rectangle with tiles to find an expression for the area. An x^2-tile has been placed to begin the process. How many x^2-tiles are needed to fill in the rectangle? What is the area of the rectangle? What is the product $(2x)(3x)$?



2. To show the product $(2x)(3x + 3)$, use algebra tiles to model $2x$ and $3x + 3$ as the dimensions of a rectangle.

a) Use two x-tiles to form one dimension along the left side. Use three x-tiles and three unit tiles to form the other dimension along the top. Complete the area of the rectangle.

b) How many x^2-tiles and x-tiles are needed? What is the resulting product?

3. Use algebra tiles to model the product $(3x)(2x + 1)$. What is the resulting product?

4. Use the same process to model the product of two binomials. What is each resulting product?

a) $(x + 1)(x + 2)$ **b)** $(x + 2)(x + 4)$

c) $(x + 3)(2x + 1)$ **d)** $(2x + 3)(x + 1)$

5. Reflect Consider your results from step 4.

a) Describe how you can use algebra tiles to multiply two binomials.

b) How are the terms in the resulting products related to the terms of the two binomials? Write a general rule for multiplying two binomials.

6. Use your rule to find each product.

a) $(x + 3)(x + 8)$ **b)** $(2x + 5)(x + 4)$

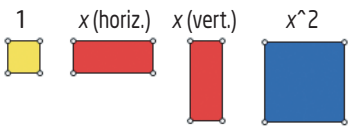
c) $(4x + 7)(3x + 1)$

Method 2: Use *The Geometer's Sketchpad*®

Tools

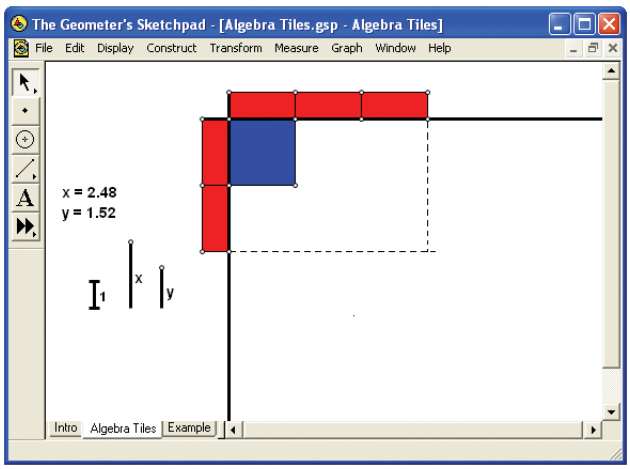
- computer with *The Geometer's Sketchpad*®
- Algebra Tiles.gsp

For this activity, you will use unit tiles, x-tiles (horizontal and vertical), and x^2-tiles.



1. To show the product $(2x)(3x)$, use virtual algebra tiles to model $2x$ and $3x$ as the dimensions of a rectangle. Then, fill in the rectangle with tiles to find an expression for the area.

- Click and hold the **Custom Tool** icon. Select **x (vertical)** and place two tiles along the vertical line, as shown.
- Repeat for **x (horizontal)**, placing three tiles along the horizontal line, as shown.
- Repeat for **x^2** (x^2-tile) to complete the area of the rectangle.



How many x^2-tiles are needed? What is the area of the rectangle? What is the product $(2x)(3x)$?

2. Model the product $2x(3x + 3)$ using virtual algebra tiles.

a) Place two **x (vertical)** tiles along the vertical line. Place three **x (horizontal)** tiles and three unit tiles along the horizontal line.

b) Complete the area of the rectangle.

c) How many x^2-tiles and x-tiles are needed? What is the resulting product?

3. Use virtual algebra tiles to illustrate the product $3x(2x + 1)$. What is the resulting product?

4. Use the same process to model the product of two binomials. What is each resulting product?

a) $(x + 1)(x + 2)$ **b)** $(x + 2)(x + 4)$

c) $(x + 3)(2x + 1)$ **d)** $(2x + 3)(x + 1)$

5. **Reflect** Consider your results from step 4.

a) Describe how you can use virtual algebra tiles to multiply two binomials.

b) How are the terms in the resulting products related to the terms of the two binomials? Write a general rule for multiplying two binomials.

6. Use your rule to find each binomial product.

a) $(x + 3)(x + 8)$ **b)** $(2x + 5)(x + 4)$

c) $(4x + 7)(3x + 1)$

Investigate B

Tools

- TI-89 calculator

How can you relate the distributive property to polynomial multiplication?

Turn on the TI-89 calculator. If necessary, press the HOME key to display the computer algebra system (CAS) Home screen.

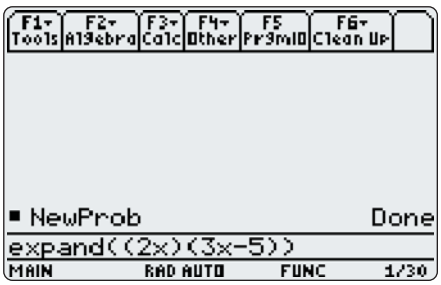
1. Clear the calculator's memory.

- Press 2nd [F6] to display the **Clean Up** menu.
- Select **2:NewProb**.
- Press ENTER.

2. Expand $2x(3x - 5)$.

- Press F2.
- Select **3:expand(**.
- Type $(2x)(3x - 5)$.
- Press) ENTER.

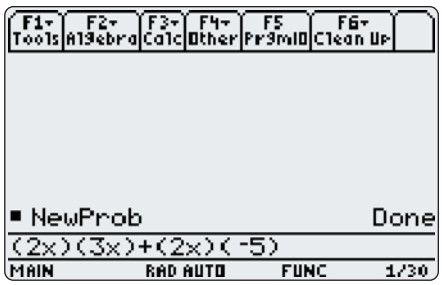
What expression results?



3. Apply the distributive property. Multiply $2x$ by each term in the binomial $(3x - 5)$.

- Type $(2x)(3x) + (2x)(-5)$.
- Press ENTER.

What expression results?



4. Compare your answers for steps 2 and 3. Describe the process the calculator used to expand the expression $2x(3x - 5)$.

5. Use a CAS to expand each expression.

a) $2x(5x^2 - 3x + 1)$

b) $-3x^2(2x^2 + 5x - 3)$

c) $3xy(2x^2y - 4xy^2)$

d) $ax(bx - cy + d)$

Technology Tip

To enter exponents, use the ^ key. For example, to enter x^2, press x ^ 2.

6. Use a CAS to apply the distributive property to expand each expression in step 5. Verify that the answers are the same.

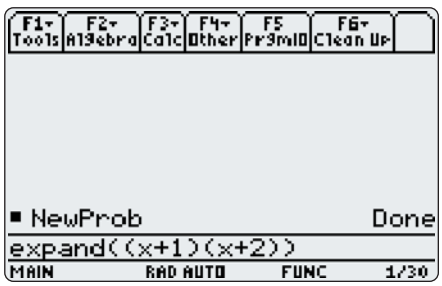
7. Use a CAS to expand the product of two binomials. What is each resulting product?

a) $(x + 1)(x + 2)$

b) $(x + 2)(x + 4)$

c) $(x + 3)(2x + 1)$

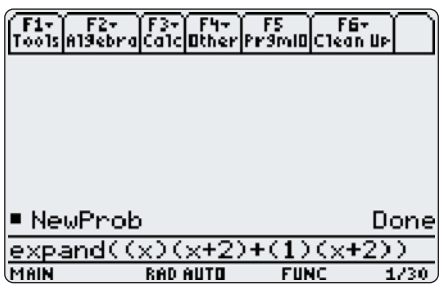
d) $(2x + 3)(x + 1)$



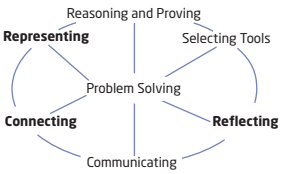
Technology Tip

To enter an expression containing variables multiplied together, you must use the × key. For example, to enter xy, press x × y.

8. Use a CAS to apply the distributive property to expand each expression in step 7. Multiply each term of the first binomial by the second binomial. Verify that the answers are the same.



9. **Reflect** Consider your results from steps 7 and 8. How are the terms in the resulting products related to the terms of the two binomials? Write a general rule for multiplying two binomials.



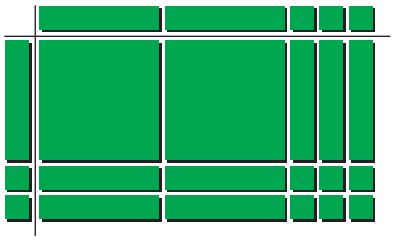
Example 1 Model a Binomial Product

Model the binomial product $(x + 2)(2x + 3)$.

Solution

Method 1: Use Algebra Tiles

Use algebra tiles to create a rectangle with width $x + 2$ and length $2x + 3$.



There are two x^2-tiles, seven x-tiles, and six unit tiles.

The area of the rectangle is $2x^2 + 7x + 6$.

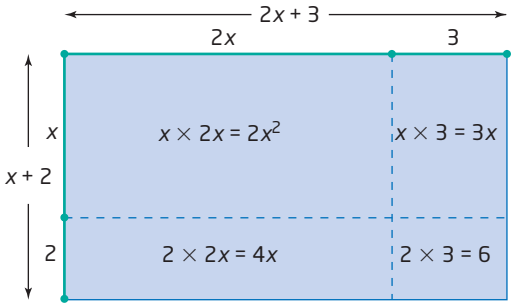
$(x + 2)(2x + 3) = 2x^2 + 7x + 6$

Method 2: Use a Diagram

Draw a vertical segment of any length and label it $x + 2$ units. Divide the segment into a section x units long and a section 2 units long.

Perpendicular to the top of the vertical segment, draw a horizontal segment, $2x + 3$ units long, divided into a section $2x$ units long and a section 3 units long.

Complete the rectangle by drawing sides opposite $x + 2$ and $2x + 3$. Then, draw horizontal and vertical dashed segments from the section marks to the opposite sides, as shown. Find the areas of the four sections that make up the whole rectangle.



The binomial product $(x + 2)(2x + 3)$ equals the sum of the four areas.

$(x + 2)(2x + 3) = 2x^2 + 3x + 4x + 6$

$= 2x^2 + 7x + 6$

Example 2 Use the Distributive Property

Find each binomial product.

a) $(x + 2)(x + 5)$

b) $(x - 2)(x + 4)$

c) $(3x + 7)(x - 5)$

Solution

a) To multiply two binomials, use the distributive property. Then, simplify by collecting like terms.

$$\begin{aligned}&(x + 2)(x + 5)\\ &= x(x + 5) + 2(x + 5)\\ &= x(x) + x(5) + 2(x) + 2(5)\\ &= x^2 + 5x + 2x + 10\\ &= x^2 + 7x + 10\end{aligned}$$

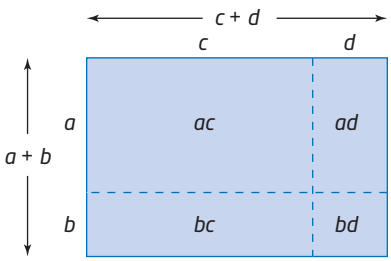
b)
$$\begin{aligned}&(x - 2)(x + 4)\\ &= x(x + 4) - 2(x + 4)\\ &= x^2 + 4x - 2x - 8\\ &= x^2 + 2x - 8\end{aligned}$$

c) Apply the distributive property mentally. Perform the multiplications indicated by the upper arrows and then the lower arrows. Then, simplify by collecting like terms.

$$\begin{aligned}&(3x + 7)(x - 5)\\ &= 3x^2 - 15x + 7x - 35\\ &= 3x^2 - 8x - 35\end{aligned}$$

You can find the product of two binomials by multiplying each term in the first binomial by each term in the second binomial. If necessary, simplify by collecting like terms.

$$(a + b)(c + d) = ac + ad + bc + bd$$



Example 3 Expand and Simplify

Expand and simplify.

a) $-2(4x - 5)(7x - 6)$

b) $2(x + 7)(x - 3) - (4x + 3)(2x - 1)$

Solution

a) First, find the product of the binomials. Then, multiply by -2.

$$-2(4x - 5)(7x - 6)$$
$$= -2(28x^2 - 24x - 35x + 30)$$
$$= -2(28x^2 - 59x + 30)$$
$$= -56x^2 + 118x - 60$$

b) $2(x + 7)(x - 3) - (4x + 3)(2x - 1)$
$$= 2(x^2 - 3x + 7x - 21) - (8x^2 - 4x + 6x - 3)$$
$$= 2(x^2 + 4x - 21) - (8x^2 + 2x - 3)$$
$$= 2(x^2 + 4x - 21) - 1(8x^2 + 2x - 3)$$
$$= 2x^2 + 8x - 42 - 8x^2 - 2x + 3$$
$$= -6x^2 + 6x - 39$$

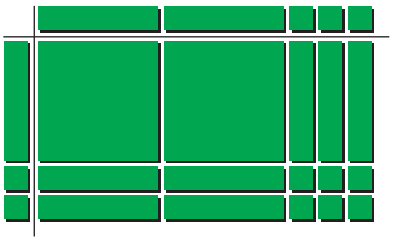
Multiplying a polynomial by –1 produces the opposite polynomial:
$(-1)(8x^2 + 2x - 3)$
$= -8x^2 - 2x + 3$

Literacy Connections

Expand, find the product, and multiply all mean the same thing.

Key Concepts

- You can model a binomial product as the area of a rectangle with the binomials as the dimensions.



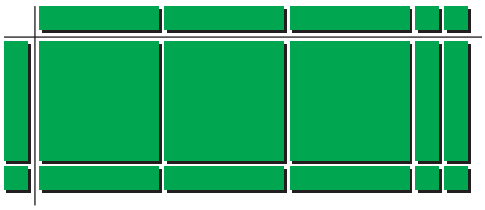
The area of the rectangle is $2x^2 + 7x + 6$.
$(x + 2)(2x + 3) = 2x^2 + 7x + 6$

- You can find the product of two binomials by multiplying each term in one binomial by each term in the other binomial. If necessary, simplify by collecting like terms.

$$(x + 2)(2x + 3)$$
$$= 2x^2 + 3x + 4x + 6$$
$$= 2x^2 + 7x + 6$$

Communicate Your Understanding

C1 Explain how the algebra tiles illustrate the product $(x + 1)(3x + 2)$.



C2 Jason expands and simplifies $(x + 3)(x + 7)$ in his head. He adds 3 and 7 to get $10x$, and multiplies 3 and 7 to get 21. His final answer is $x^2 + 10x + 21$.

a) Explain Jason's logic.

b) Would Jason's method work when expanding $(2x + 3)(5x + 6)$? If yes, explain why. If no, how should he adapt his method?

c) Would this method work for negative constant terms? Explain.

C3 Describe the steps you would use to expand $(3x + 5)(2x - 9)$.

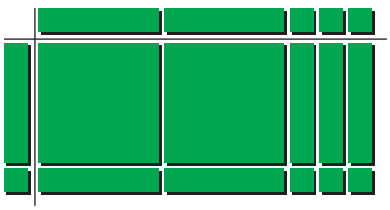
C4 Rolly uses an acronym to help him remember how to expand two binomials. It is called the FOIL method, for *First*, *Outside*, *Inside*, *Last*. What does this mean?

Practise

For help with questions 1 and 2, see Example 1.

1. What binomial product does each model illustrate?

a)



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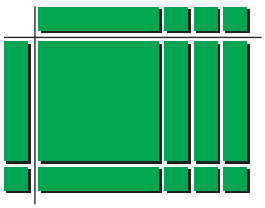
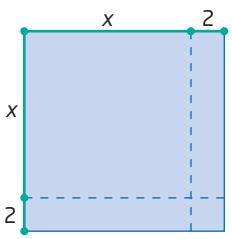
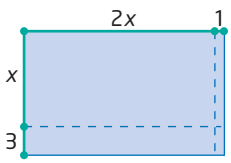


c)



d)



2. Model each binomial product using algebra tiles, virtual algebra tiles, or a diagram.

a) $(2x + 1)(x + 1)$ **b)** $(x + 4)(x + 2)$

c) $(x + 1)(x + 5)$ **d)** $(2x + 1)(3x + 2)$

For help with questions 3 to 6, see Example 2.

3. Use the distributive property to find each binomial product.

a) $(x + 3)(x + 5)$ **b)** $(x + 3)(x + 4)$

c) $(y + 2)(y + 4)$ **d)** $(r + 4)(r + 2)$

e) $(n + 7)(n + 1)$ **f)** $(p + 9)(p + 9)$

g) $(w + 7)(w + 8)$ **h)** $(d + 3)(d + 11)$

4. Use the distributive property to find each binomial product.

a) $(k - 3)(k - 5)$ **b)** $(y - 3)(y - 4)$

c) $(x - 2)(x - 4)$ **d)** $(q - 4)(q - 2)$

e) $(j - 7)(j - 1)$ **f)** $(p - 9)(p - 3)$

g) $(z - 7x)(z - 8x)$ **h)** $(b - 3c)(b - 11c)$

5. Use the distributive property to find each binomial product.

a) $(x + 3)(x - 5)$ **b)** $(y + 3)(y - 4)$

c) $(c - 2)(c + 4)$ **d)** $(w - 4)(w + 2)$

e) $(m + 7)(m - 1)$ **f)** $(y - 9)(y + 3)$

g) $(x + 7y)(x - 8y)$ **h)** $(a + 6b)(a - 10b)$

6. Find each binomial product.

a) $(2x + 3)(x + 4)$ **b)** $(y - 3)(5y - 7)$

c) $(6c - 1)(3c + 5)$ **d)** $(7w - 2)(2w + 1)$

e) $(5m + 6)(5m - 6)$

f) $(9y - 2)(2y + 2)$

g) $(7d + 5c)(8d - 6c)$

h) $(6q + 5r)(7q - 12r)$

For help with questions 7 and 8, see Example 3.

7. Expand and simplify.

a) $3(x - 5)(x + 6)$ **b)** $-2(x - 7)(x - 9)$

c) $-(y + 2)(y - 8)$ **d)** $2(k + 3)(k + 7)$

e) $m(m - 3n)(m - 5n)$

f) $2a(3a + 4b)(6a + 7b)$

8. Expand and simplify.

a) $(x + 4)(x + 6) + (x - 1)(x + 7)$

b) $(2x + 5)(3x - 7) + 2(4x + 9)(2x - 11)$

c) $3(6x - 2)(6x - 1) - (2x - 3)(5x + 6)$

d) $-(x - 2)(x - 3) + 2(3x + 5)(x + 4)$

e) $(x + 4)^2 - (x - 4)^2$

f) $-5(3x - 1)(5x - 2) + 6(6x + 3)(5x - 2)$

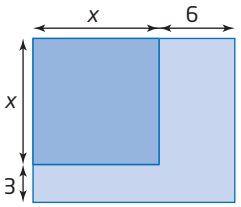
Connect and Apply

9. The predicted flight path of a firework is defined by the relation $h = -2(d - 3)(d - 15)$, where d is the horizontal distance, in metres, from a safety wall, and h is the height, in metres.

a) Expand and simplify the relation.

b) Verify that the relation from part a) is equivalent to the original relation. Use both relations to determine the height of the firework if d represents 10 m.

10. A square garden has side length x. One dimension is increased by 6 m and the other is increased by 3 m.



a) Write an algebraic expression for the area of the original garden.

b) Write an algebraic expression for the area of the new garden.

c) Expand and simplify your area expression from part b).

d) Find an expression that represents the increase in area.

e) If x represents 12 m, find the increase in area.

11. a) For each situation, begin with a square field, measuring x metres by x metres. Then, draw a diagram of the new field, write an algebraic expression for its area, and expand and simplify your area expression.

i) The length of one side is increased by 10 m.

ii) The length of one side is doubled.

iii) The length is increased by 5 m and the width is increased by 6 m.

b) **Use Technology** Use a CAS to verify your answers to part a).

12. A parabola has equation $y = (x + 3)(x - 1)$.

a) Find the x-intercepts of $y = (x + 3)(x - 1)$.

b) Expand and simplify the equation.

c) Graph the result from part b). Verify that the x-intercepts are the same.

Making Connections

In Chapter 4, you worked with quadratic relations of the form $y = (x - r)(x - s)$.

13. A rectangular prism has width w centimetres, length 2 cm more than its width, and height 2 cm.

a) Draw a diagram of the prism.

b) Express the volume as a product.

c) Expand and simplify the volume expression.

14. A cube has side length x. Each dimension is increased by y.

a) Draw a diagram of the cube.

b) Write an algebraic expression for the surface area of the original cube.

c) Write an algebraic expression for the surface area of the new cube.

d) Write an algebraic expression for the difference in surface area. Expand and simplify.

e) Write an algebraic expression for the difference in volume. Expand and simplify.

15. A scuba diver is drifting in a current of 0.3 m/s. If she swims with the current at an additional speed of v metres per second, the distance, d, in metres, that she travels before running out of air can be modelled by the relation $d = 3000(v + 0.3)(1.0 - v)$.

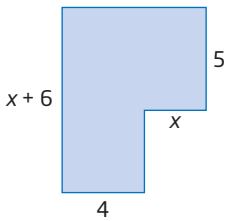
a) Expand this relation.

b) If she swims at 0.2 m/s, how far can she swim before running out of air?

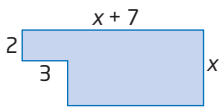
c) **Use Technology** Use a graphing calculator or computer software to investigate what swimming speed would result in the maximum distance before running out of air.

16. Write an algebraic expression for the area of each figure. Expand and simplify. Then, find the area in another way to verify your result.

a)



b)



Extend

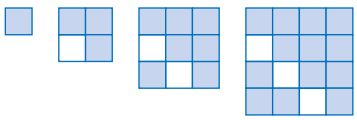
17. The number of hot dogs, n, sold by Wayne's Wiener World on a given day is modelled by $n = 500 - 100p$, where p is the price, in dollars.

a) Solve this equation for p.

b) The revenue generated by hot dog sales is $R = np$. Substitute your expression for p from part a), and expand to obtain an expression for the daily hot dog revenue.

c) **Use Technology** Use a CAS to verify your answer to part b).

18. Determine an algebraic expression for the number of shaded small squares in the nth diagram. Test your expression for two more diagrams.



19. Pose and solve a problem in which the dimensions of a rectangular object are written as binomials.

20. **Math Contest** When a positive integer n is divided by 24, the remainder is 18. When n is divided by 8 the remainder is

A 0 **B** 1 **C** 2 **D** 4 **E** 6