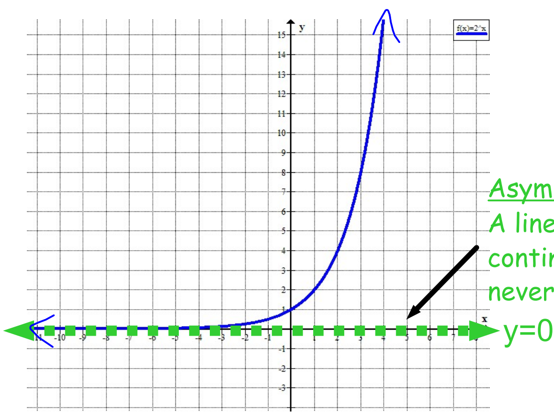
3.1 Exponential Growth and Decay

Exponential growth



Asymptote:
A line that a curve will continually approach but never touch

Exponential Decay

Notice: The graph is increases slowly then quickly (slope becomes steeper)

Notice: The graph is decreasing quickly then slowly (slope becomes less steep)

The graph of an exponential growth or decay is a smooth curve that is almost horizontal at one end (approaches an asymptote) and rapidly increases or decreases at the other end.

The equation of an exponential relation contains a constant base and a variable exponent. ex.

$$A = 250(1.04)^{3x}$$

$$y = -3(5)^{x}$$

$$T = t(0.76)^{\frac{3}{4}w}$$

Exponential growth or decay can be modelled by an exponential equation:

# of growth/decay periods

Final Amount (Amt. after "x" growth/decay periods)

$$\boxed{\text{Note: } x = \frac{t}{d} \quad or \quad x = \frac{t}{h}}$$

Where d=time it takes to double

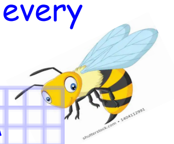
h=time it takes to divide in half

$$A = a_o\,(b)^x$$

Amount at beginning.

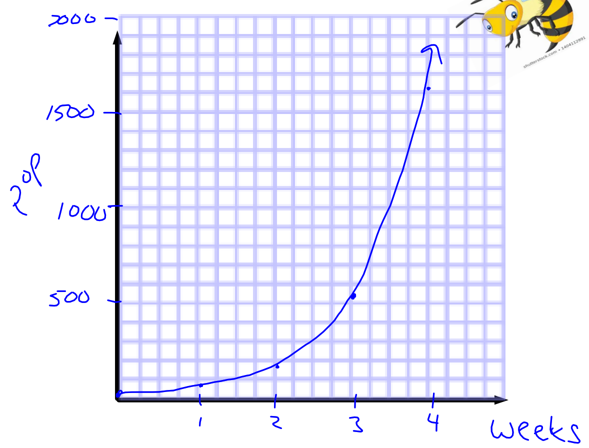
growth factor (b > 1)
decay factor (0< b < 1)

Ex 1 A wasp population starts at an initial population of 20 and triples every week.



a) Complete the table and graph.

Week	Population	First Diff	Second Diff
0	20		
		40	
1	60		80
		120	
2	180		240
		360	
3	540		720
		1080	
4	1620		



b) Look for a pattern in the population and differences. What do you notice?

They have a common ratio!

Pull

The first and second differences are NOT common however they have a constant multiplier of 3. The y values also have the same multiplier.

c) Find an equation to model this growth. HINT: Use the constant multiplier/ common factor.

Let x be # weeks

$$A = a_0 b^x$$
$$= 20(3)^x$$

$a_0 \rightarrow$ initial amount

$b \rightarrow$ growth factor

d) Use the equation to find the number of wasps after one year.

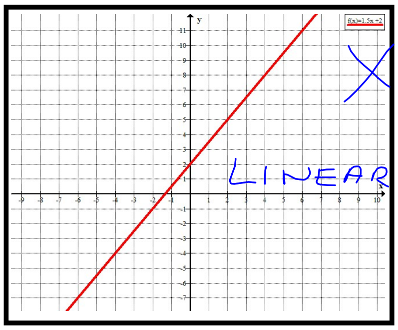
$$A = 20(3)^x$$
$$= 20(3)^{52}$$
$$= 1.3 \times 10^{26}$$

Weeks in a year?

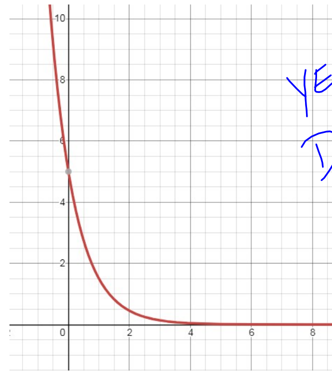
52

$\therefore$ There will be 1.3×10^{26} wasps in a year

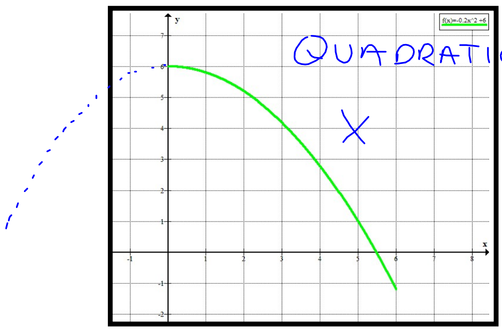
Ex. 2 Which could represent exponential growth or decay?



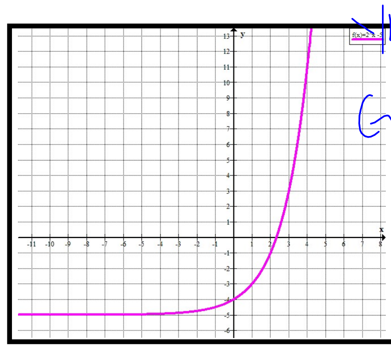
X LINEAR



YES Decay



QUADRATIC X



YES Growth

$y = 5x^2 + 7x - 3$

NO

$A = 400\,(\underline{0.76})^{t/4}$

YES $b < 1$ $\therefore$ Decay

$P = 200(\underline{1.07})^t$

YES $\hookrightarrow b > 1$ $\therefore$ growth

x	y
0	1
1	5
2	25
3	125
4	625
5	3125

×5 ×5 ×5 ×5

YES! Growth

t	A
0	80
1	72
2	64.8
3	58.32
4	52.49
5	47.24

$\frac{72}{80} = 0.9$

$\frac{64.8}{72} = 0.9$

×0.9 ×0.9

YES! Decay

Ex. 3 The table below shows the amount of radioactive material remaining from a 300 g sample.

Time (hours)	Amount (g)
0	300
1	285
2	270.75
3	257.21
4	244.35
5	232.13

ratio of "y" values

0.95
0.95
0.95
0.95
0.95

a) Write an exponential equation to model the situation.

$A = a_0 b^x$

$= 300(0.95)^x$

Let x represent time (hours)

b) Determine an approximate growth/decay rate.

Factor → 0.95

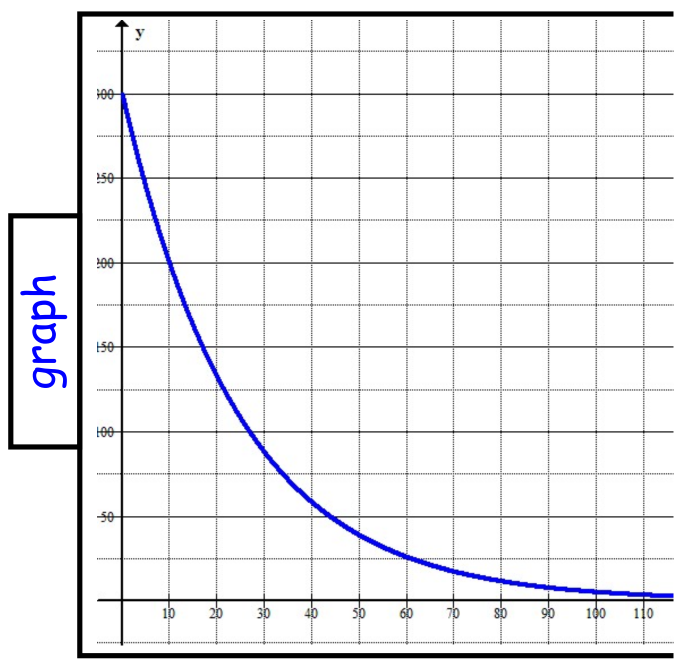
Rate → 5% $(1-0.95) \times 100\%$

c) Use this equation to determine the amount that will remain after 12 hours.

$A = 300(0.95)^{12}$

$\doteq 162.1$

∴ There will be approx. 162.1g remaining



Ex. 4 Model each situation with an exponential equation.
Define "x" for each.

a) A car worth $25 000, depreciates in value by 13% each year.

$a_0 = 25000$ Let x rep. # of years

$b = 0.87$

$A = 25000(0.87)^x$

rate → 13% (0.13)

factor → 0.87

b) 400 mg of radioactive material deteriorates by 5% every 4 hours.

$a_0 = 400$ Let x represent # of 4 hour chunks.

$b = 0.95$

$A = 400(0.95)^x$

Let x represent # hours

$A = 400(0.95)^{\frac{x}{4}}$

c) A rabbit population of 50 doubles every 6 weeks.

$a_0 = 50$ Let x reprrsent # weeks

$b = 2$

$A = 50(2)^{\frac{x}{6}}$

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Hmk.

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