3.3 - Rational Exponents

when the exponent is a rational number.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

index: indicates what root you want

Evaluate $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$9^{\frac{1}{2}} = \sqrt{9} = 3$

Note: when the index is 2 we don't write it... it is understood (square root)

Ex 1. Evaluate.

a) $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

b) $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

c) $(-16)^{\frac{1}{2}} = \sqrt{-16}$

∴ NO REAL ROOTS!

d) $-16^{\frac{1}{2}} = -16^{\frac{1}{2}} = -\sqrt{16} = -4$

e) $16^{\frac{-1}{4}}$

$= (16^{\frac{1}{4}})^{-1} = (\sqrt[4]{16})^{-1} = 2^{-1} = \frac{1}{2}$

$= (16^{-1})^{\frac{1}{4}} = \left(\frac{1}{16}\right)^{\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

numerator is the exponent

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

denominator is the index

OR

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Ex 2. Evaluate.

a) $27^{\frac{2}{3}}$

$= \left(\sqrt[3]{27}\right)^2$

$= 3^2$

$= 9$

b) $81^{\frac{5}{4}}$

$= \left(\sqrt[4]{81}\right)^5$

$= 3^5$

$= 243$

c) $16^{\frac{-3}{2}}$

$= \frac{1}{16^{\frac{3}{2}}}$

$= \frac{1}{\left(\sqrt{16}\right)^3}$

$= \frac{1}{64}$

d) $-8^{\frac{2}{3}}$

$= -\left(\sqrt[3]{8}\right)^2$

$= -4$

e) $(-27)^{\frac{-1}{3}}$

$= \left(\sqrt[3]{-27}\right)^{-1}$

$= (-3)^{-1}$

$= \frac{1}{-3}$

$= -\frac{1}{3}$

f) $\left(\frac{4}{81}\right)^{\frac{-3}{2}}$

$= \left(\frac{81}{4}\right)^{\frac{3}{2}}$

$= \frac{\left(\sqrt{81}\right)^3}{\left(\sqrt{4}\right)^3}$

$= \frac{729}{8}$

g) $32^{0.4}$

$0.4 = \frac{10}{10}$

$= \frac{4}{10}$

$= \frac{2}{5}$

$= 32^{\frac{2}{5}}$

$= \left(\sqrt[5]{32}\right)^2$

$= 2^2$

$= 4$

h) $25^{\frac{4}{9}} \cdot 5^{\frac{1}{9}}$

HINT! need to change the base

$= \left(5^2\right)^{\frac{4}{9}} \cdot 5^{\frac{1}{9}}$

$= 5^{\frac{8}{9}} \cdot 5^{\frac{1}{9}}$

$= 5^{\frac{9}{9}}$

$= 5$

Ex 3. Write the following radicals in exponent form.

a) $\sqrt[5]{7x^4}$

$= (7x^4)^{\frac{1}{5}}$

$= 7^{\frac{1}{5}} x^{\frac{4}{5}}$

b) $\sqrt[3]{\sqrt{x^5}}$

$= \left((x^5)^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$= x^{\frac{5}{6}}$

c) $\sqrt[3]{-5x^4}$

$= (-5x^4)^{\frac{1}{3}}$

$= (-5)^{\frac{1}{3}} x^{\frac{4}{3}}$

d) $\dfrac{1}{\sqrt[7]{x^9}}$

$= \dfrac{1}{x^{\frac{9}{7}}}$

$= x^{-\frac{9}{7}}$

f) $\left(\sqrt[3]{x^3y^2}\right)\left(\sqrt[4]{x^{-2}y^3}\right)$

$= (x^3y^2)^{\frac{1}{3}}(x^{-2}y^3)^{\frac{1}{4}}$

$= x^{\frac{3}{3}} y^{\frac{2}{3}} \cdot x^{-\frac{2}{4}} y^{\frac{3}{4}}$

$= x^{1-\frac{1}{2}} y^{\frac{2}{3}+\frac{3}{4}}$

$= x^{\frac{1}{2}} y^{\frac{17}{12}}$

g) $\left(\sqrt[5]{2a^3b^4c^{-2}}\right)^4$

$= \left(\left(2a^3b^4c^{-2}\right)^{\frac{1}{5}}\right)^4$

$= \left(2a^3b^4c^{-2}\right)^{\frac{4}{5}}$

$= 2^{\frac{4}{5}} a^{\frac{12}{5}} b^{\frac{16}{5}} c^{-\frac{8}{5}}$

$= \dfrac{2^{\frac{4}{5}} a^{\frac{12}{5}} b^{\frac{16}{5}}}{c^{\frac{8}{5}}}$

h) $\sqrt[5]{m^3} \bullet \sqrt[4]{m^5}$

$= (m^3)^{\frac{1}{5}} \cdot (m^5)^{\frac{1}{4}}$

$= m^{\frac{3}{5}} \cdot m^{\frac{5}{4}}$

$= m^{\frac{37}{20}}$

$\frac{3}{5} + \frac{5}{4}$

$= \frac{12}{20} + \frac{25}{20}$

$= \frac{37}{20}$

Homework

Handout -->3.3