3.6 Solving Exponential Equations

Method: Convert to a common base using exponent rules, then solve.

- Make both sides be the same base
- If you have nothing but a power on either side, if their bases are equal, their exponents must be equal.

Example 1: Solve

a) $2^x = 32$

$2^x = 2^5$

$\therefore x = 5$

b) $(-4)^x = -1024$

$(-4)^x = (-4)^5$

$\therefore x = 5$

c) $-3^y = -81$

$(-1) \quad (-1)$

$3^y = 81$

$3^y = 3^4$

$\therefore y = 4$

d) $5^{4-3x} = 25^{x+1}$

$5^{4-3x} = (5^2)^{x+1}$

$5^{4-3x} = 5^{2x+2}$

$\therefore 4 - 3x = 2x + 2$

$2 = 5x$

$\frac{2}{5} = x$

e) $3^{3x+1} = 1$

$3^{3x+1} = 3^0$

$\therefore 3x + 1 = 0$

$3x = -1$

$x = -\frac{1}{3}$

f) $4^{x+5} = 8^{1-3x}$

$(2^2)^{x+5} = (2^3)^{1-3x}$

$2^{2x+10} = 2^{3-9x}$

$\therefore 2x+10 = 3-9x$

$11x = -7$

$x = -\frac{7}{11}$

g) $7^{3-x} = \frac{1}{49}$

$7^{3-x} = 7^{-2}$

$\therefore 3-x = -2$

$5 = x$

$= \frac{1}{7^2}$

$= 7^{-2}$

Now it's your turn!

h) $\frac{1}{256} = 4^{5x+1}$

$4^{-4} = 4^{5x+1}$

$\therefore -4 = 5x+1$

$x = -1$

i) $9^{2x+3} = 27^{\frac{x}{4}}$

$(3^2)^{2x+3} = (3^3)^{\frac{x}{4}}$

$3^{4x+6} = 3^{\frac{3x}{4}}$

$\therefore 4x+6 = \frac{3x}{4}$

$16x+24 = 3x$

$13x = -24$

$x = \frac{-24}{13}$

j) $\frac{4(3^{5x-1})}{4} = \frac{36}{4}$ 9

$3^{5x-1} = 9$

$3^{5x-1} = 3^2$

$\therefore 5x-1 = 2$

$x = \frac{3}{5}$

Sometimes, we must find a common factor before solving using a common base.

Example 2: Solve

a) $2^{x+4} + 2^x = 136$

$2^x 2^4 + 2^x = 136$

$2^x(2^4 + 1) = 136$

$2^x(17) = 136$

$2^x = 8$

$2^x = 2^3$

$\therefore x = 3$

b) $3^{x+1} - 3^{x+4} = -702$

$3^x \cdot 3 - 3^x 3^4 = -702$

$3^x(3 - 3^4) = -702$

$3^x(-78) = -702$

$3^x = 9$

$3^x = 3^2$

$\therefore x = 2$

You try...

c) $7^{x+1} + 7^{x+2} = 392$

$7^x \cdot 7 + 7^x \cdot 7^2 = 392$

$7^x(7 + 7^2) = 392$

$7^x(56) = 392$

$7^x = 7$

$\therefore x = 1$

Homework: Handout