3.7 Applications of Exponential Functions

Recall:

Amount after "x" growth/decay periods

$$A = a_o\,(b)^x$$

# of growth/decay periods

Amount at beginning.

growth factor (b > 1)
decay factor (0< b < 1)

Note: To calculate the number of growth/decay periods, divide the *total time* by the *Doubling* or *Halving time* (i.e. the growth/decay time).

$x = \dfrac{t}{d}$ or $x = \dfrac{t}{h}$

Ex. 1 A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 cells initially, how many will there be in 3 hours?

Givens

$a_o = 350$
$b = 2$

$t = ?$
$t = 3$ hours
$= 180$ min

$A = a_o b^{\frac{t}{d}}$
$= 350(2)^{\frac{t}{20}}$
$= 350(2)^{\frac{180}{20}}$
$= 179\,200$

$\therefore$ There will be 179 200 cells.

Ex. 2 The half-life of a radioactive element is 15 days.
a) Write a function relating the amount remaining, in grams, to the time, in days.
b) How much of a 200 gram sample will be left after 150 days?

a) $A = a_o\left(\frac{1}{2}\right)^{\frac{t}{15}}$

"Half-life" means $b = \frac{1}{2}$

b) $a_o = 200$
$t = 150$

$A = 200\left(\frac{1}{2}\right)^{\frac{150}{15}}$
$\doteq 0.195$

$\therefore$ There will be approx. 0.195g

Ex. 3 In 2001, the population of Canada was 31 051 000. What is the percent growth, if the current population of Canada is 40 431 811 ?

Givens

$A = 40\,431\,811$

$t = 25$ years (2026 - 2001)

$a_0 = 31\,051\,000$

$A = a_0 b^x$

$40\,431\,811 = 31\,051\,000\, b^{25}$

$1.302109 = b^{25}$

$b = \sqrt[25]{1.302109}$

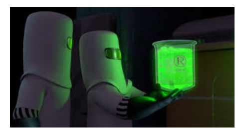
$\doteq 1.0106$

means 101% ⇒ Grew by 1% each year

Ex. 4 A radioactive substance has a half life of 2.4 days.

a) What <u>fraction</u> of the original amount would remain after 12 days?

b) How long would it take until only 12.5% of the original amount remained?



a) $A = a_0\left(\frac{1}{2}\right)^{\frac{t}{2.4}}$

$t = 12$? $a_0 = 1$

$A = \left(\frac{1}{2}\right)^{\frac{12}{2.4}}$

$= \left(\frac{1}{2}\right)^5$

$= \frac{1}{32}$

∴ The remaining fraction is $\frac{1}{32}$

b) Use fake numbers!

$a_0 = 100$, $A = 12.5$

OR

easier! → $a_0 = 1000$, $A = 125$

$A = a_0\left(\frac{1}{2}\right)^{\frac{x}{2.4}}$

$125 = 1000\left(\frac{1}{2}\right)^{\frac{x}{2.4}}$

$\frac{125}{1000} = \left(\frac{1}{2}\right)^{\frac{x}{2.4}}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{x}{2.4}}$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{x}{2.4}}$

∴ $3 = \frac{x}{2.4}$

$7.2 = x$

∴ It would take 7.2 days

Ex. 5 256g of a substance decays to 64g in 15.6 hours. Determine the half-life of the substance.

$\Lambda \ \lambda$

$A = a_0\left(\frac{1}{2}\right)^{\frac{t}{d}}$ ← Finding this!

$64 = 256\left(\frac{1}{2}\right)^{\frac{15.6}{d}}$

$\frac{64}{256} = \left(\frac{1}{2}\right)^{\frac{15.6}{d}}$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{15.6}{d}}$

$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{\frac{15.6}{d}}$

$\therefore\ 2 = \frac{15.6}{d}$

$d = \frac{15.6}{2}$

$= 7.8$

Givens

$t = 15.6$ hrs

$a_0 = 256$ g

$b = \frac{1}{2}$

$A = 64$

$\therefore$ The half-life is 7.8 hours

HOMEWORK
Handout