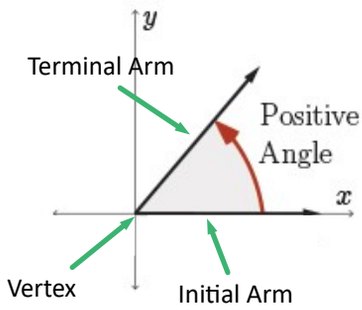
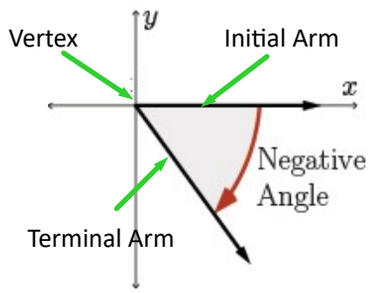
Lesson 4.1: Angles between 0^0 and 360^0

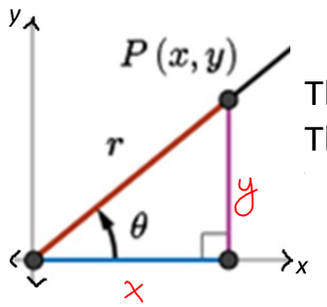
Terminology

An angle is in standard position when its vertex is at the origin and its initial arm is on the positive x-axis.





Consider the point P(x,y) on the terminal arm of angle θ in standard position.



The side opposite to θ is y.
The side adjacent to θ is x.

$r^2 = x^2 + y^2$

can find using Pythagorean Theorem

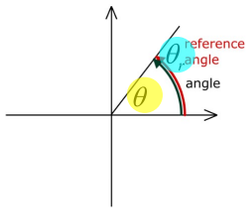
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Principal Angle θ

The angle measured between the initial arm and the terminal arm.

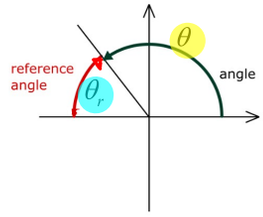
Reference Angle or Related Acute Angle θ_r

The angle between the terminal arm and the x-axis ($\theta_r < 90^0$).



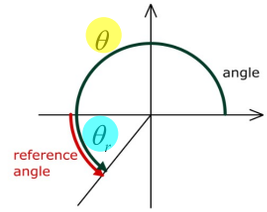
I quadrant

$\theta_r = \theta$



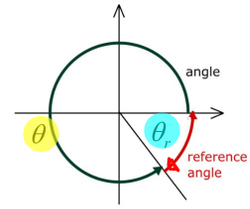
II quadrant

$\theta_r = 180 - \theta$



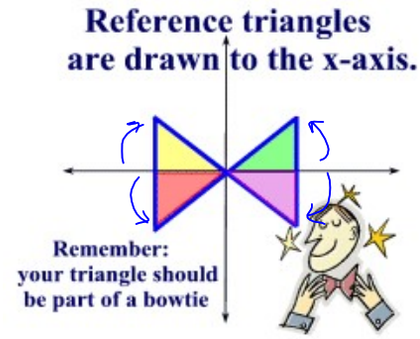
III quadrant

$\theta_r = \theta - 180$



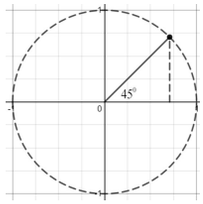
IV quadrant

$\theta_r = 360 - \theta$



Exploring the primary trigonometric ratios on a coordinate grid.

Terminal arm is in Quadrant 1

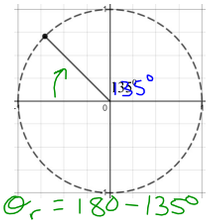


$\sin\theta = 0.7071$

$\cos\theta = 0.7071$

$\tan\theta = 1$

Terminal arm is in Quadrant 2



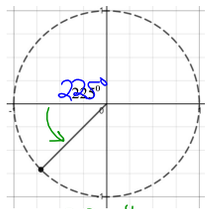
$\sin\theta = 0.7071$

$\cos\theta = -0.7071$

$\tan\theta = -1$

$\theta_r = 180 - 135^\circ$
$= 45^\circ$

Terminal arm is in Quadrant 3



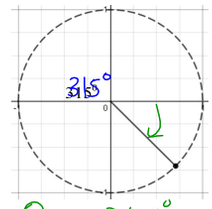
$\sin\theta = -0.7071$

$\cos\theta = -0.7071$

$\tan\theta = 1$

$\theta_r = 225^\circ - 180^\circ$
$= 45^\circ$

Terminal arm is in Quadrant 4



$\sin\theta = -0.7071$

$\cos\theta = 0.7071$

$\tan\theta = -1$

$\theta_r = 360^\circ - 315^\circ$
$= 45^\circ$

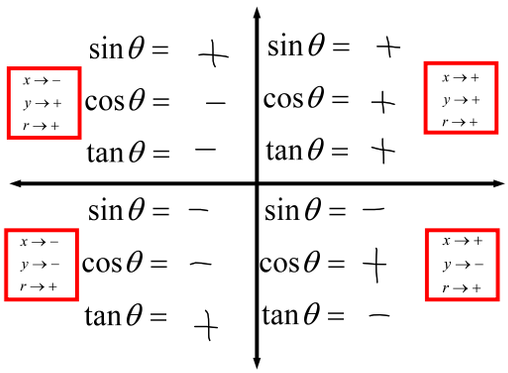
- For each of the above examples determine the reference/related acute angle, θ_r

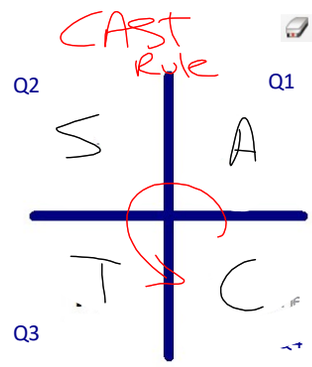
- Evaluate

$\sin 45 =$ $\sin 135 =$ $\sin 225 =$ $\sin 315 =$

What do you notice?

The CAST rule

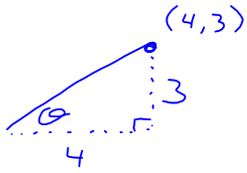




Ex 1: Given the following points on the terminal arm of θ, determine $\sin\theta$, $\cos\theta$, $\tan\theta$, and θ.

a) $P(4,3)$

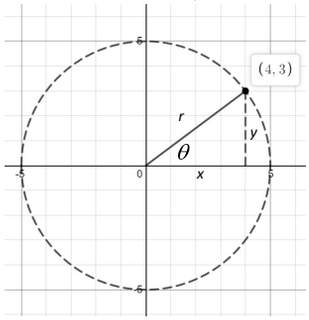
$x = 4$
$y = 3$
$r = 5$



$r = \sqrt{4^2+3^2}$
$= 5$

Quadrant I

Note : $\theta = \theta_r$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

Solve for θ :

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

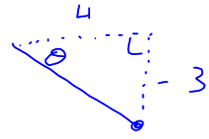
$= 37^\circ$

$\theta = \theta_r$

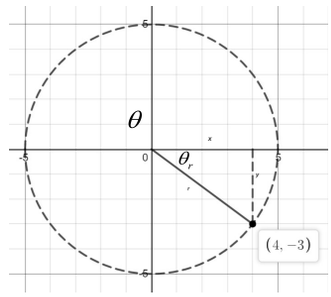
$\therefore \theta = 37^\circ$

b) $P(4,-3)$

$x = 4$
$y = -3$
$r = 5$



Quadrant IV



$\sin\theta = -\frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = -\frac{3}{4}$

Solve for θ :

$\cos\theta = \frac{4}{5}$

$\theta_r = 37^\circ$

$\theta = 360^0 - \theta_r$

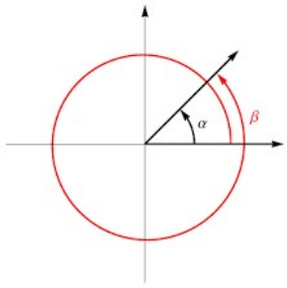
$\therefore \theta = 360 - 37^\circ$

$= 323^\circ$

Ex. 2: Evaluate, to four decimal places.

a) $\cos 154^0$ $\doteq -0.8988$

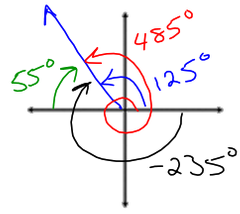
b) $\tan 230^0$ $\doteq 1.1918$



Co-terminal Angles:

- Angles with the same terminal arm.
- They can be positive or negative.

Ex. 3: Draw an angle of 125° in standard position.



a) What is the principal angle? $\theta =$ 125° (180 − 125)

b) What is the related acute angle? $\theta_r =$ 55°

c) Determine a positive angle that is co-terminal $\theta =$ 485° (360 + 125)

d) Determine a negative angle that is co-terminal $\theta =$ −235° (− (180 + 55))

Ex. 4: Determine another angle, with a different terminal arm, that would have the same trig ratios as the one given. Include a diagram.

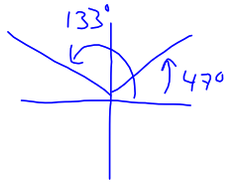
a) $\sin 47^0$

$\theta_r = 47°$

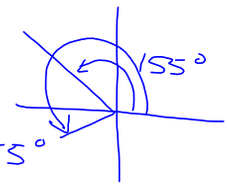
If we choose sin

$\theta = 180° - 47°$
$= 133°$

$\therefore \sin 133°$



b) $\cos 155^0$



$\theta_r = 180 - 155°$
$= 25°$

If we choose cos

$\theta = 180 + 25°$
$= 205°$

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