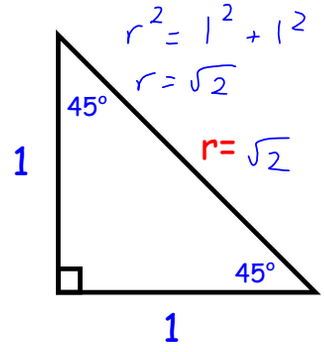
4.2 Special Angles & The Unit Circle

Special Angles:

a) 45° (isosceles triangle)



$r^2 = 1^2 + 1^2$

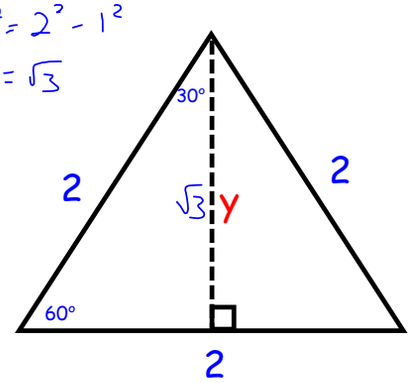
$r = \sqrt{2}$

$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$

b) 30°, 60° (half an equilateral triangle)



$y^2 = 2^2 - 1^2$

$y = \sqrt{3}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

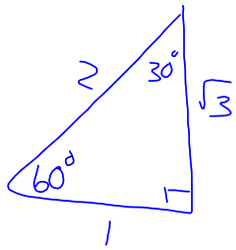
$\cos 60^\circ = \frac{1}{2}$

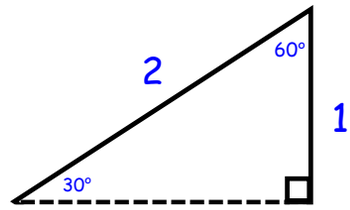
$\tan 60^\circ = \sqrt{3}$

$\sin 30^\circ = \frac{1}{2}$

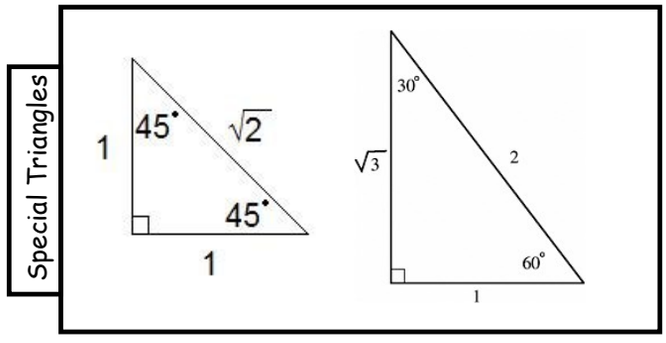
$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

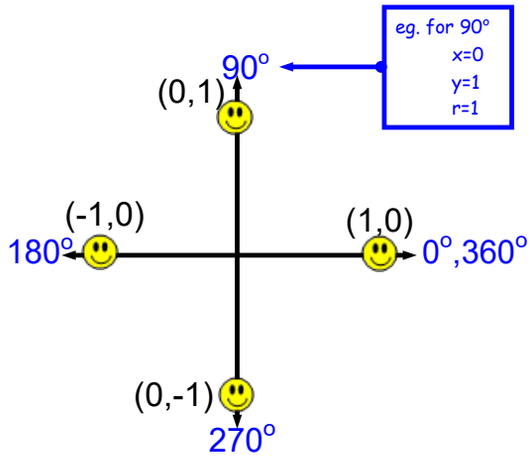




Memorize this



c) 0°,90°,180°,270°,360° (terminal arm lies on the x-axis or y-axis)

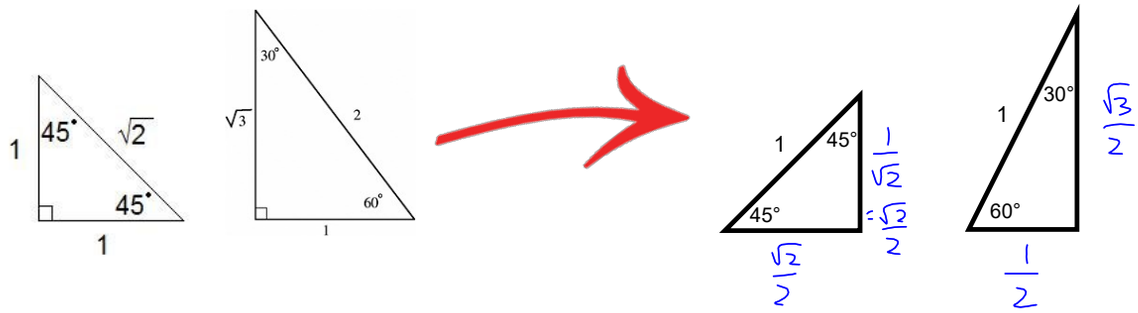


assume r=1 (...of course we'll explain later)

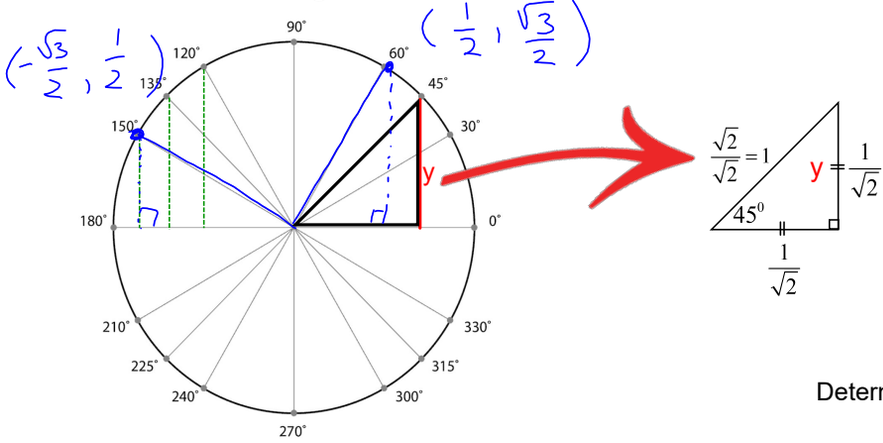
	0°	90°	180°	270°	360°
sinθ	0	1	0	-1	0
cosθ	1	0	-1	0	1
tanθ	0	undefined	0	undefined	0

The unit circle:

The unit circle is a way to "standardize" the ratios of the special angles onto one diagram.

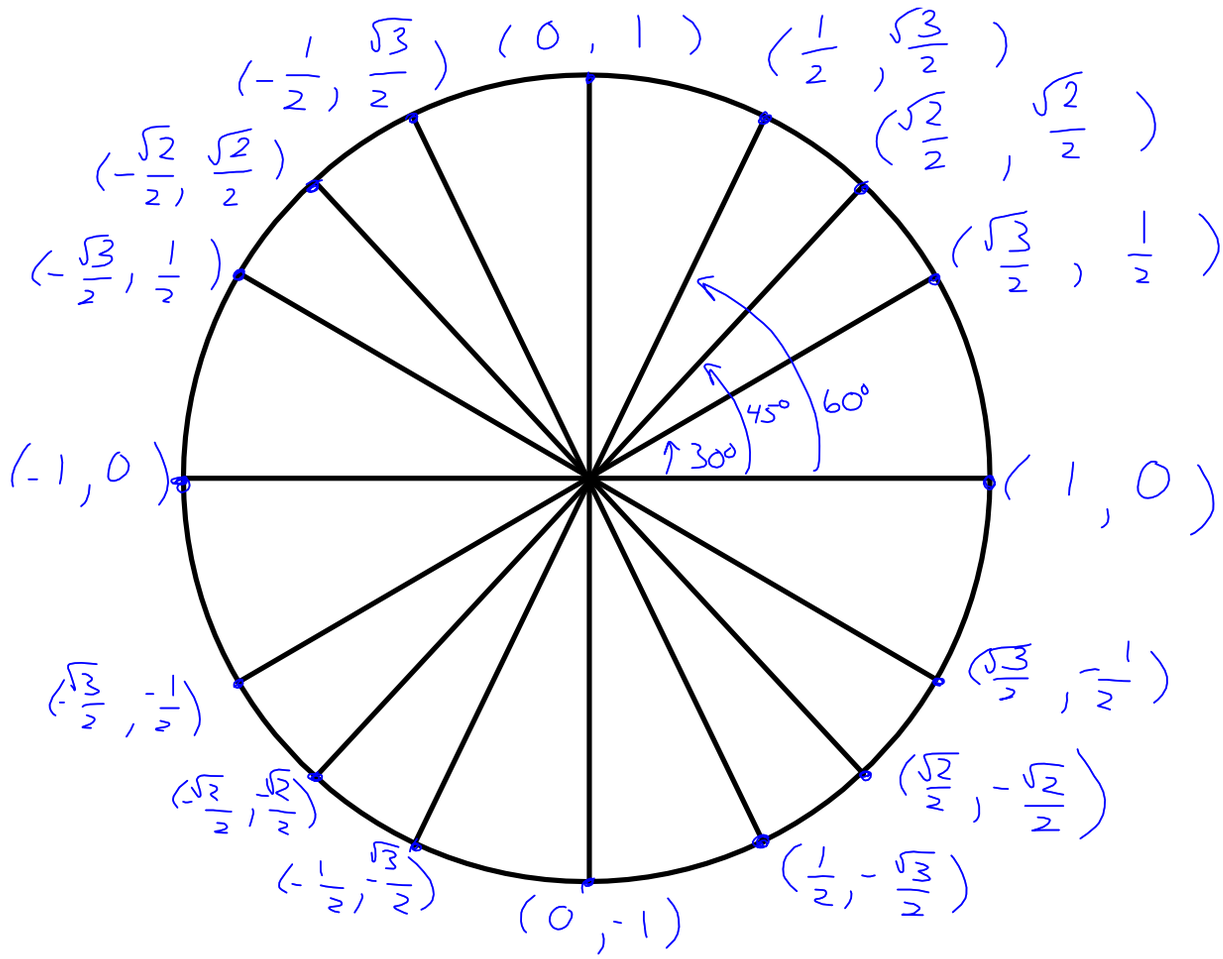


Consider these triangles on a circle with radius of 1 unit.

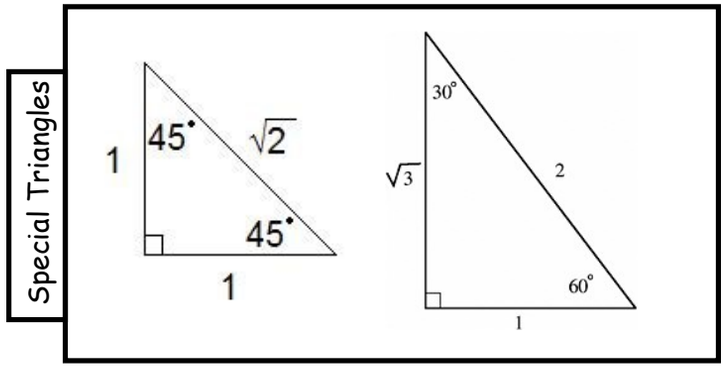


Determine y: $\sin 45 = \frac{y}{1}$

NOTE: The coordinates are $(\cos 45^0, \sin 45^0)$.



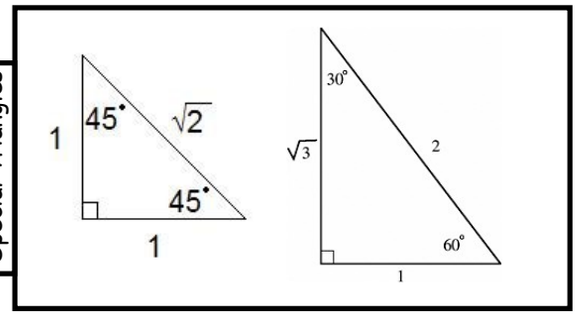
NOTE: The special values are the coordinates! (cos θ, sin θ)



Ex. 1 Determine the exact values of each tria. ratio.

Steps:
1. Determine the quadrant.
2. Draw diagram & find related angle.
3. Use special angles to find ratio.
4. Use CAST rule to determine sign (+ or -).

Special Triangles



a) $\cos 60°$

$= \frac{1}{2}$

b) $\sin 45°$

$= \frac{\sqrt{2}}{2}$

c) $\tan 30°$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

All in Q1 ∴ Positive

d) $\sin 240°$

$\theta_r = 60°$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin 240° = -\frac{\sqrt{3}}{2}$

e) $\tan 300°$

$\theta_r = 60°$

$\tan 60° = \sqrt{3}$

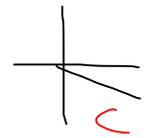
$\tan 300° = -\sqrt{3}$

tan is NEG

f) $\cos 330°$

$\theta_r = 30°$

$\cos 330° = \frac{\sqrt{3}}{2}$



cos is POS

g) $\tan 135°$

$\theta_r = 45°$

$\tan 45° = 1$

$\tan 135° = -1$

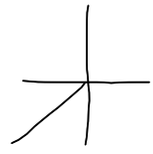
TAN is NEG

h) $\cos 225°$

$\theta_r = 45°$

$\cos 45° = \frac{\sqrt{2}}{2}$

$\cos 225° = -\frac{\sqrt{2}}{2}$



COS NEG

i) $\sin 150°$

$\theta_r = 30°$

$\sin 150° = \frac{1}{2}$

Ex. 2 Determine all possible values for $0 \le \theta \le 360°$.

Steps
1. Determine possible quadrants.
2. Draw diagram with terminal arms.
3. Determine related angle.
4. Find angles of rotation to each terminal arm.

Special Triangles

a) $\sin A = \frac{1}{\sqrt{2}}$

$\theta_r = 45°$

S | A
T | C

Q1: $\theta = 45°$

Q2: $\theta = 180 - 45 = 135°$

$\theta = 45°, 135°$

b) $\cos A = \frac{1}{2}$

$\theta = 60°$

S | A
T | C

Q1: $\theta = 60°$

Q4: $\theta = 360 - 60 = 300$

$\therefore \theta = 60°, 300°$

c) $\tan A = 1$

$\theta_r = 45°$

S | A
T | C

Q1: $\theta = 45°$

Q3: $\theta = 180 + 45 = 225°$

$\theta = 45°, 225°$

d) $\cos A = \frac{-1}{\sqrt{2}}$

e) $\tan A = -\sqrt{3}$

f) $\cos A = \frac{-\sqrt{3}}{2}$

Ex 3: Evaluate the following. Use exact, values if appropriate.

$$\sin 30^0 \cos^2\left(225^0\right) - \tan\left(-60^0\right)$$

$\cos^2\theta = (\cos\theta)^2$

$= \sin 30^\circ (\cos 225^\circ)^2 - \tan(-60^\circ)$

$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)^2 - (-\sqrt{3})$

$= \left(\frac{1}{2}\right)\left(\frac{2}{4}\right) + \sqrt{3}$

$= \frac{1}{4} + \sqrt{3}$

$= \frac{1+4\sqrt{3}}{4}$

Homework:
Handout 4.2A - Circled Questions