

Lesson 4.3 Reciprocal Trigonometric Ratios

In every right triangle, there are three primary trig ratios and three reciprocal trig ratios.

The reciprocal trig ratios are:

cosecant
reciprocal of sin

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} \\ &= \frac{1}{\left(\frac{y}{r}\right)} \\ &= \frac{r}{y} \end{aligned}$$

secant
reciprocal of cos

$$\begin{aligned} \sec \theta &= \frac{1}{\cos \theta} \\ &= \frac{1}{\left(\frac{x}{r}\right)} \\ &= \frac{r}{x} \end{aligned}$$

cotangent
reciprocal of tan

$$\begin{aligned} \cot \theta &= \frac{1}{\tan \theta} \\ &= \frac{1}{\left(\frac{y}{x}\right)} \\ &= \frac{x}{y} \end{aligned}$$

Ex. 1 Find the exact values of the following.

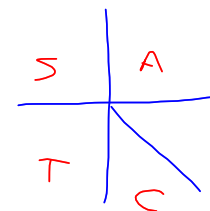
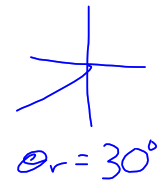
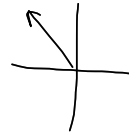
a) $\sec 45^\circ$
 $\cos 45^\circ = \frac{1}{\sqrt{2}}$
 $\sec 45^\circ = \frac{\sqrt{2}}{1}$
 $= \sqrt{2}$

b) $\cot 120^\circ$
 $-\tan 120^\circ$
 $= -\tan 60^\circ$
 $= -\sqrt{3}$
 $\cot 120 = -\frac{1}{\sqrt{3}}$
 $= -\frac{\sqrt{3}}{3}$

c) $\csc 210^\circ$
 $\sin 210^\circ$
 $= -\sin 30^\circ$
 $= -\frac{1}{2}$
 $\csc 210^\circ = -2$

d) $\csc 180^\circ$
 $\sin 180^\circ = 0$
 $\csc 180^\circ = \frac{1}{0}$
 $= \text{UNDEFINED!}$

e) $\cot 675^\circ$
 $\tan 675^\circ$
 $= \tan 315^\circ$
 $= -\tan 45^\circ$
 $= -1$
 $\cot 675^\circ = -1$



The Unit Circle

Special Triangles

Ex.2 Find the exact values of θ for $0 \leq \theta \leq 360^\circ$.

a) $\sec \theta = -\frac{2}{\sqrt{3}}$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\theta_r = 30^\circ$$

(S)	A
(T)	C

$$\begin{array}{l} Q_2 \\ \theta = 180 - 30 \\ = 150 \end{array}$$

$$\begin{array}{l} Q_3 \\ \theta = 180 + 30 \\ = 210 \end{array}$$

$$\therefore \theta = 150^\circ, 210^\circ$$

b) $\csc \theta = -\sqrt{2}$

$$\sin \theta = -\frac{1}{\sqrt{2}}$$

$$\theta_r = 45^\circ$$

$$\begin{array}{l} Q_3 \\ \theta = 180 + 45 \\ = 225^\circ \end{array}$$

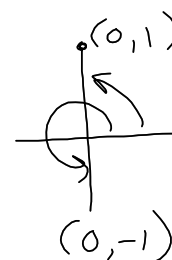
$$\begin{array}{l} Q_4 \\ \theta = 360 - 45 \\ = 315^\circ \end{array}$$

$$\therefore \theta = 225^\circ, 315^\circ$$

c) $\cot \theta = 0$

$$\tan \theta = \frac{1}{0}$$

undefined!



$$\theta = 90^\circ, 270^\circ$$

Ex.3 Evaluate the following to four decimal places. Use calculators!

$$\begin{aligned} \text{a) } \csc 67^\circ &= \frac{1}{\sin 67^\circ} \\ &\approx 1.0864 \end{aligned}$$

$$\boxed{x^{-1}} \quad \boxed{1/x}$$

$$\text{d) } \sec(-104^\circ)$$

$$\begin{aligned} &= \frac{1}{\cos(-104^\circ)} \\ &\approx -4.1336 \end{aligned}$$

$$\begin{aligned} \text{b) } \cot 342^\circ &= \frac{1}{\tan 342^\circ} \\ &\approx -3.0777 \end{aligned}$$

$$\begin{aligned} \text{c) } \sec 143^\circ &= \frac{1}{\cos 143^\circ} \\ &\approx -1.2521 \end{aligned}$$

Ex. 4 Determine the value of θ for $0^\circ \leq \theta \leq 360^\circ$. Use your calculators. Round to the nearest degree.

a) $\csc \theta = -1.7842$

$$\sin \theta = -\frac{1}{1.7842}$$

$$\theta_r = \sin^{-1}\left(\frac{1}{1.7842}\right)$$

$$\approx 34^\circ$$

S	A
(T)	(C)

$\frac{Q_3}{\theta = 180 + 34}$
 $= 214^\circ$

$\frac{Q_4}{\theta = 360 - 34}$
 $= 326^\circ$

$$\theta = 214^\circ, 326^\circ$$

b) $\cot \theta = -0.4663$

$$\tan \theta = -\frac{1}{0.4663}$$

$$\theta_r = \tan^{-1}\left(\frac{1}{0.4663}\right)$$

$$\approx 65^\circ$$

$\frac{Q_2}{\theta = 180 - 65}$
 $= 115^\circ$

$\frac{Q_4}{\theta = 360 - 65}$
 $= 295^\circ$

$$\therefore \theta = 115^\circ, 295^\circ$$

(S)	A
T	(C)

Pg. 280 # 4, 5, 6

Pg. 304 # 1, 2, 3, 6, 13