

Review Practice: Exponential Functions

1. Expand and simplify each expression.

a) $(-3xy)^4$ b) $\frac{6x^3y^6}{3x^2yz^3}$ c) $\left(\frac{y}{3}\right)^{-4}$ d) $2(x^2y)^3 \cdot 3xy^{-3}$
 e) $3x^{-4}$ f) $\left(\frac{2x}{3y^2}\right)^{-2}$ g) $\frac{x^2y^{-4}z^{-2}}{p^{-1}q^3}$ h) $\frac{1}{2^{-4}}$

2. Evaluate the following.

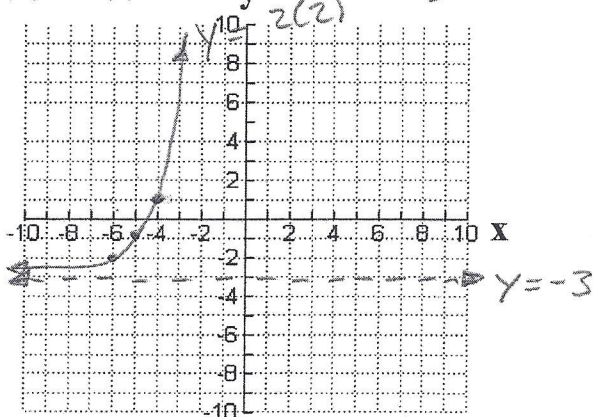
a) $8^{\frac{4}{3}}$ b) $64^{\frac{2}{3}}$ c) $(-125)^{\frac{5}{3}}$ d) $(4x^2)^{\frac{3}{2}}$

3. Solve each equation for x.

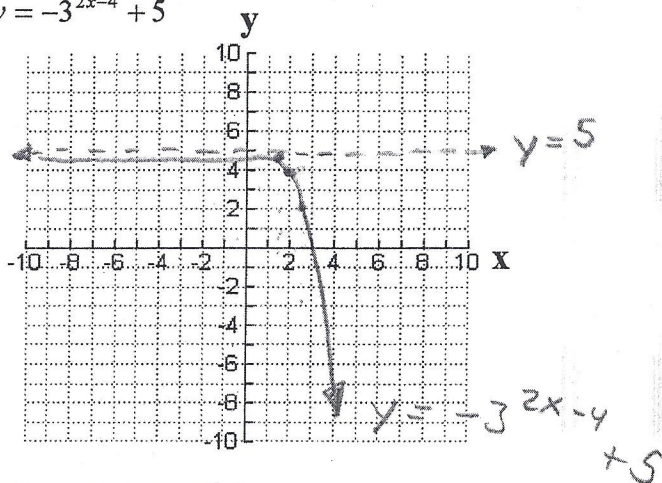
a) $2^{2x+1} = 8$ b) $16^{x+1} = 2^{x+13}$ c) $2^{3x} = \frac{1}{64}$ d) $2^{2x} - 6(2^x) = 16$

4. Graph the following exponential functions and state their domain and range.

a) $y = 2(2)^{x+5} - 3$



b) $y = -3^{2x-4} + 5$



5. Determine if each relationship is linear, quadratic or exponential:

a)

x	y
3	6
5	22
7	46
9	78
11	118

1st diff, 2nd diff.

quadratic

x	y
0	0
1	2
2	8
3	26
4	80

1st diff ratio

exponential

x	y
-4	15
-2	20
0	25
2	30
4	35

1st diff.

linear

6. a) Yuri purchases a new GM Puma for \$23500 in 2010. The car depreciates at a rate of 21% each year. How much is the car worth when Yuri sells it in 2016?

~~b) During early spring, the thickness of ice on Guelph Lake drops in half every 10 days. If there is currently 15 cm of ice on the lake, when will the thickness be 5 cm?~~

Review Practice: Exponential Functions

$$1a) (-3xy)^4 \\ = (-3)^4 x^4 y^4 \\ = 81x^4 y^4$$

$$b) \frac{6x^3 y^6}{3x^2 y z^3} \\ = \frac{2xy^5}{z^3}$$

$$c) \left(\frac{y}{3}\right)^{-4} \\ = \left(\frac{3}{y}\right)^4 \\ = \frac{81}{y^4}$$

$$d) 2(x^2 y)^3 \cdot 3xy^{-3} \\ = 2x^6 y^3 \cdot \frac{3x}{y^3} \\ = 6x^7$$

$$e) 3x^{-4} \\ = \frac{3}{x^4}$$

$$f) \left(\frac{2x}{3y^2}\right)^{-2} \\ = \left(\frac{3y^2}{2x}\right)^2 \\ = \frac{9y^4}{4x^2}$$

$$g) \frac{x^2 y^{-4} z^{-2}}{y^{-1} z^3} \\ = \frac{x^2 p}{y^4 z^2 q^3}$$

$$h) \frac{1}{2^{-4}} \\ = 2^4 \\ = 16$$

$$2.a) 8^{\frac{4}{3}} \\ = (8^{\frac{1}{3}})^4 \\ = (2)^4 \\ = 16$$

$$b) 64^{\frac{2}{3}} \\ = (64^{\frac{1}{3}})^2 \\ = (4)^2 \\ = 16$$

$$c) (-125)^{\frac{5}{3}} \\ = (-125^{\frac{1}{3}})^5 \\ = (-5)^5 \\ = -3125$$

$$d) (4x^2)^{\frac{3}{2}} \\ = (\sqrt{4x^2})^3 \\ = (2x)^3 \\ = 8x^3$$

$$3a) 2^{2x+1} = 8 \\ 2^{2x+1} = 2^3 \\ 2x+1 = 3 \\ 2x = 3-1 \\ 2x = 2 \\ \frac{2x}{2} = \frac{2}{2} \\ x = 1$$

$$b) 16^{x+1} = 2^{x+13} \\ (2^4)^{x+1} = 2^{x+13} \\ 2^{4x+4} = 2^{x+13} \\ 4x+4 = x+13 \\ 4x-x = 13-4 \\ 3x = 9 \\ \frac{3x}{3} = \frac{9}{3} \\ x = 3$$

$$c) 2^{3x} = \frac{1}{64} \\ 2^{3x} = 2^{-6} \\ 3x = -6 \\ \frac{3x}{3} = \frac{-6}{3} \\ x = -2$$

$$\text{extend } d) 2^{2x} - 6(2^x) = 16 \\ (2^x)^2 - 6(2^x) - 16 = 0 \\ \text{Let } n = 2^x \\ n^2 - 6n - 16 = 0 \\ (n-8)(n+2) = 0 \\ n = 8 \text{ or } n = -2$$

Case 1

$$n = 2^x$$

$$8 = 2^x$$

$$2^3 = 2^x$$

$$x = 3$$

Case 2

$$-2 = 2^x$$

No solⁿ

$$y = 2(2)^{x+5} - 3$$

4. a)

x	y = 2 ^x
-1	1/2
0	1
1	2

$$k = 1$$

$$d = -5$$

$$a = 2$$

$$c = -3$$

axis of symmetry:
y = -3

$$\text{Domain} = \{x \mid x \in \mathbb{R}\}$$

$$\text{Range} = \{y \mid y > -3, y \in \mathbb{R}\}$$

Graph → see handout

b)

$$y = -3^{2x-4} + 5$$

$$y = -1(3)^{2(x-2)} + 5$$

axis of symmetry:
y = 5

x	y = 3 ^x
-1	1/3
0	1
1	3

$$k = 2$$

$$p = 2$$

$$a = -1$$

$$q = 5$$

$$\text{Domain} = \{x \mid x \in \mathbb{R}\}$$

$$\text{Range} = \{y \mid y < 5, y \in \mathbb{R}\}$$

Graph → see handout

5. See handout.

6. a)

$$y = ab^x$$

$$y = 23500(0.79)^x$$

$$y = 23500(0.79)^6$$

$$y = \$5712.55$$

$$b = 1 - r$$

$$= 1 - 0.21$$

$$= 0.79$$

~~6. b)~~

$$y = a \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$y = 15(0.5)^{\frac{t}{10}}$$

$$5 = 15(0.5)^{\frac{t}{10}}$$

$$\frac{1}{3} = 0.5^{\frac{t}{10}}$$

Using natural logarithms

$$\log\left(\frac{1}{3}\right) = \log\left(0.5^{\frac{t}{10}}\right)$$

$$\frac{\log(1/3)}{\log(0.5)} = \frac{t}{10} \frac{\log(0.5)}{\log(0.5)}$$

$$\frac{1.58}{1} = \frac{t}{10}$$

$$t \approx 15.8 \text{ days}$$

trial error

$$0.5^{1.74} = 0.5^{\frac{t}{10}}$$

$$\frac{1.58}{1} = \frac{t}{10}$$

$$t \approx 15.8 \text{ days}$$