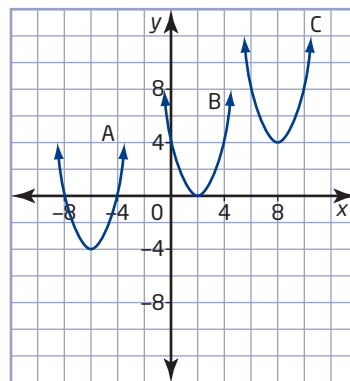


# 6.4

## The Quadratic Formula

Quadratic equations that can be factored are fairly simple to solve. But what about quadratics that cannot be factored? The Greek mathematicians Euclid (300 BCE) and Pythagoras (500 BCE) both derived geometric solutions to a quadratic equation. A general solution for quadratic equations, using numbers, was derived in about 700 AD by the Hindu mathematician Brahmagupta. The general formula used today was derived in about 1100 AD by another Hindu mathematician, Bhaskara. He was also the first to recognize that any positive number has two square roots, one positive and one negative.



For each parabola shown, how many real roots does the related quadratic equation have?



### Investigate A

**How can you use the process of completing the square to solve quadratic equations?**

1. Graph the relation  $y = x^2 - 9$ . How many  $x$ -intercepts are there?
2. **a)** Solve the equation  $x^2 - 9 = 0$  by isolating  $x^2$  first.
  - b)** How many roots are there?
  - c)** How do the roots relate to the graph in step 1?
3. **a)** Graph the relation  $y = (x + 2)^2 - 9$ . How many  $x$ -intercepts are there?
  - b)** Solve the equation  $(x + 2)^2 - 9 = 0$  by isolating the perfect square,  $(x + 2)^2$ , then taking the square root of both sides, and finally solving for  $x$ . Remember that there are two square roots.
  - c)** How many roots are there? How do the roots relate to the graph in part a)?
4. **a)** Solve the equation  $x^2 + 10x + 16 = 0$  by first completing the square.
  - b)** Describe the steps you used to solve for  $x$ .

5. **Reflect** Would the method you used to solve the quadratic equation in step 4 work for any quadratic equation? Explain.
6. Use your method to solve each quadratic equation.
- a)  $3x^2 + 30x + 48 = 0$
- b)  $5x^2 - 20x - 52 = 0$

## Investigate B

### How can you use completing the square to derive a formula for solving quadratic equations?

By completing the square, you can develop a formula that can be used to solve any quadratic equation. Copy the calculations in the table. Describe the steps in the example and how they relate to the development of the **quadratic formula**.

Example	Quadratic Formula
$2x^2 + 5x + 1 = 0$	$ax^2 + bx + c = 0$
$x^2 + \frac{5}{2}x + \frac{1}{2} = 0$	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
$\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) + \frac{1}{2} = 0$	$\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + \frac{c}{a} = 0$
$\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right) - \left(\frac{5}{4}\right)^2 + \frac{1}{2} = 0$	$\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$
$\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{1}{2} = 0$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$
$\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{8}{16} = 0$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2} = 0$
$\left(x + \frac{5}{4}\right)^2 - \frac{17}{16} = 0$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$
$\left(x + \frac{5}{4}\right)^2 = \frac{17}{16}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
$x + \frac{5}{4} = \pm\sqrt{\frac{17}{16}}$	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$
$x + \frac{5}{4} = \pm\frac{\sqrt{17}}{4}$	$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{5}{4} \pm \frac{\sqrt{17}}{4}$	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-5 \pm \sqrt{17}}{4}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### quadratic formula

- a formula for determining the roots of a quadratic equation of the form  $ax^2 + bx + c = 0$ ,  $a \neq 0$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## Example 1 Real Roots

Use the quadratic formula to solve each quadratic equation. Where necessary, round to the nearest hundredth.

Verify graphically using technology.

a)  $2x^2 + 9x + 6 = 0$

b)  $4x^2 - 12x = -9$

### Solution

a) For  $2x^2 + 9x + 6 = 0$ ,  $a = 2$ ,  $b = 9$ , and  $c = 6$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-9 \pm \sqrt{9^2 - 4(2)(6)}}{2(2)} \\&= \frac{-9 \pm \sqrt{81 - 48}}{4} \\&= \frac{-9 \pm \sqrt{33}}{4}\end{aligned}$$

The exact roots are  $\frac{-9 + \sqrt{33}}{4}$  and  $\frac{-9 - \sqrt{33}}{4}$ .

You can also express the answers as approximate roots.

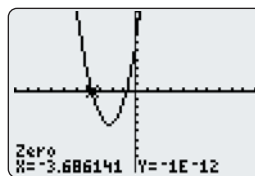
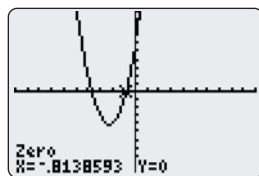
$$\begin{aligned}x &= \frac{-9 + \sqrt{33}}{4} & \text{or} & & x &= \frac{-9 - \sqrt{33}}{4} \\&\doteq -0.81 & & & &\doteq -3.69\end{aligned}$$

```
(-9+√(33))/4  
-.8138593384  
(-9-√(33))/4  
-3.686140662
```

The approximate roots are  $-0.81$  and  $-3.69$ , to the nearest hundredth.

Use the Zero operation of a graphing calculator to verify that the roots are the zeros of the related quadratic relation

$$y = 2x^2 + 9x + 6.$$



### Technology Tip

When a graphing calculator displays a result such as  $-1E-12$ , it represents a number in scientific notation.

$$-1E-12 = -1 \times 10^{-12}$$

b) First, write  $4x^2 - 12x = -9$  in the form  $ax^2 + bx + c = 0$ .

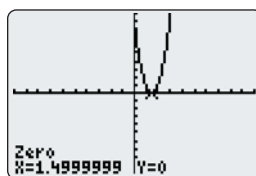
$$4x^2 - 12x + 9 = 0$$

For  $4x^2 - 12x + 9 = 0$ ,  $a = 4$ ,  $b = -12$ , and  $c = 9$ .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)} \\ &= \frac{12 \pm \sqrt{144 - 144}}{8} \\ &= \frac{12 \pm \sqrt{0}}{8} \\ &= \frac{12}{8} \\ &= \frac{3}{2} \end{aligned}$$

The root is  $\frac{3}{2}$  or 1.5.

I can see that the trinomial on the left side of the equation is a perfect square, so I could have solved by factoring.



When the x-intercepts are known, you can find the x-coordinate of the vertex by finding the midpoint of the line segment connecting the x-intercepts.

Use the two x-intercepts from the quadratic formula.

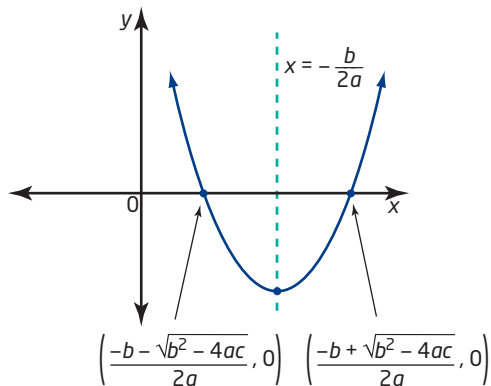
$$\begin{aligned} x &= \frac{\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}}{2} \\ &= \frac{\frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}}{2} \\ &= \frac{-2b}{4a} \\ &= -\frac{b}{2a} \end{aligned}$$

The square root expressions have a zero sum.

$$\sqrt{b^2 - 4ac} - \sqrt{b^2 - 4ac} = 0$$

The x-coordinate of the vertex is  $-\frac{b}{2a}$ .

This also gives the equation of the axis of symmetry,  $x = -\frac{b}{2a}$ .



## Example 2 Use the Quadratic Formula to Sketch a Parabola

Find the  $x$ -intercepts, the vertex, and the equation of the axis of symmetry of the quadratic relation  $y = -5x^2 + 8x - 3$ . Sketch the parabola.

### Solution

To find the  $x$ -intercepts, let  $y = 0$  and use the quadratic formula to solve the quadratic equation.

For  $-5x^2 + 8x - 3 = 0$ ,  $a = -5$ ,  $b = 8$ , and  $c = -3$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-8 \pm \sqrt{8^2 - 4(-5)(-3)}}{2(-5)} \\&= \frac{-8 \pm \sqrt{64 - 60}}{-10} \\&= \frac{-8 \pm \sqrt{4}}{-10} \\&= \frac{-8 \pm 2}{-10}\end{aligned}$$

Therefore,

$$\begin{aligned}x &= \frac{-8 - 2}{-10} \quad \text{or} \quad x = \frac{-8 + 2}{-10} \\&= 1 \quad \quad \quad = \frac{3}{5} \text{ or } 0.6\end{aligned}$$

The  $x$ -intercepts are 0.6 and 1.

To find the  $x$ -coordinate of the vertex, use  $x = -\frac{b}{2a}$ .

$$\begin{aligned}x &= -\frac{8}{2(-5)} \\&= -\frac{8}{-10} \\&= \frac{4}{5} \text{ or } 0.8\end{aligned}$$

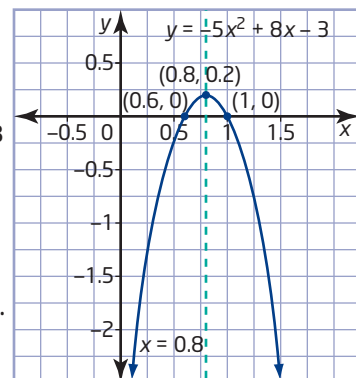
I can check this one mentally. The  $x$ -coordinate of the midpoint of the line segment connecting the  $x$ -intercepts, 0.6 and 1, is 0.8.

Substitute  $x = 0.8$  into  $y = -5x^2 + 8x - 3$  to find the  $y$ -coordinate of the vertex.

$$\begin{aligned}y &= -5(0.8)^2 + 8(0.8) - 3 \\&= -3.2 + 6.4 - 3 \\&= 0.2\end{aligned}$$

The coordinates of the vertex are  $(0.8, 0.2)$ .

The axis of symmetry has equation  $x = 0.8$ .



### Example 3 Connect a Parabola and No Real Roots

A parabola has equation  $y = (x - 2)^2 + 3$ .

- State the coordinates of the vertex, the equation of the axis of symmetry, and the direction of opening.
- Determine the  $x$ -intercepts. Verify using the quadratic formula.
- Sketch the parabola.

#### Solution

- The vertex is  $(2, 3)$ . The equation of the axis of symmetry is  $x = 2$ . The parabola opens upward, since  $a$  is positive.
- Since the vertex of the parabola is above the  $x$ -axis and it opens upward, it has no  $x$ -intercepts. This can be verified using the quadratic formula after expanding and simplifying the original equation.

$$\begin{aligned}y &= (x - 2)^2 + 3 \\ &= x^2 - 4x + 4 + 3 \\ &= x^2 - 4x + 7\end{aligned}$$

Let  $y = 0$  and use the quadratic formula with  $a = 1$ ,  $b = -4$ , and  $c = 7$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16 - 28}}{2} \\ &= \frac{4 \pm \sqrt{-12}}{2}\end{aligned}$$

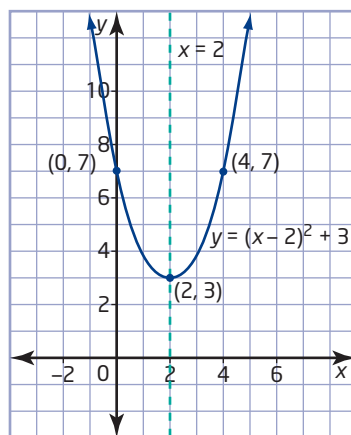
Since the square root of a negative number is not a real number, there are no real roots.

Therefore, the parabola has no  $x$ -intercepts.

- Let  $x = 0$ . A second point on the curve is  $(0, 7)$ .

Looking at the  $x$ -coordinate of this point, I know that it is located 2 units to the left of the axis of symmetry. So, its partner is located 2 units to the right of the axis of symmetry and has the same  $y$ -coordinate.

Then, due to symmetry, the partner point on the parabola is  $(4, 7)$ .



### Example 4 Path of a Basketball

The path of a basketball after it is thrown from a height of 1.5 m above the ground is given by the equation

$h = -0.25d^2 + 2d + 1.5$ ,  
where  $h$  is the height, in metres, and  $d$  is the horizontal distance, in metres.

- How far has the ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?
- Find the horizontal distance when the basketball is at a height of 4.5 m above the ground.



### Solution

- When the basketball lands on the ground, the height is 0 m. Let  $h = 0$ .  
For  $-0.25d^2 + 2d + 1.5 = 0$ ,  $a = -0.25$ ,  $b = 2$ , and  $c = 1.5$ .

$$\begin{aligned}d &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-2 \pm \sqrt{2^2 - 4(-0.25)(1.5)}}{2(-0.25)} \\&= \frac{-2 \pm \sqrt{4 + 1.5}}{-0.5} \\&= \frac{-2 \pm \sqrt{5.5}}{-0.5}\end{aligned}$$

So,  $d \doteq -0.7$  or  $d \doteq 8.7$ .

Since  $d$  represents distance, it must be positive.

The basketball has travelled a horizontal distance of about 8.7 m when it lands on the ground.

```
(-2+sqrt(5.5))/-0.5
-.6904157598
(-2-sqrt(5.5))/-0.5
8.69041576
```

b) Let  $h = 4.5$ .

$$-0.25d^2 + 2d + 1.5 = 4.5$$

$$-0.25d^2 + 2d - 3 = 0$$

For  $-0.25d^2 + 2d - 3 = 0$ ,  $a = -0.25$ ,  $b = 2$ , and  $c = -3$ .

$$\begin{aligned} d &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(-0.25)(-3)}}{2(-0.25)} \\ &= \frac{-2 \pm \sqrt{4 - 3}}{-0.5} \\ &= \frac{-2 \pm \sqrt{1}}{-0.5} \\ &= \frac{-2 \pm 1}{-0.5} \end{aligned}$$

```

(-2+1)/-0.5      2
(-2-1)/-0.5      6
  
```

So,  $d = 2$  or  $d = 6$ .

The basketball will be at a height of 4.5 m twice along its parabolic path: on the way up at a horizontal distance of 2 m and on the way down at a horizontal distance of 6 m.

I need to express the equation in the form  $ax^2 + bx + c = 0$ .

## Key Concepts

- A quadratic equation of the form  $ax^2 + bx + c = 0$ ,  $a \neq 0$ , can be solved for  $x$  using the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .
- The  $x$ -coordinate of the vertex of a parabola is  $-\frac{b}{2a}$ , and the equation of the axis of symmetry is  $x = -\frac{b}{2a}$ .

Solving Quadratic Equations:  $ax^2 + bx + c = 0$

Method	When It Can Be Used	Comments
graphing	always	The solutions will not always be exact: this is best used only when an approximate answer is needed.
factoring	sometimes	Use when $c = 0$ or when factors are easily found.
completing the square	always	This is best used for equations of the form $x^2 + bx + c = 0$ , where $b$ is an even number.
quadratic formula	always	This method always gives exact solutions, but in some cases the other methods are easier to use.
computer algebra system (CAS)	always	Use the solve (or factor) function of the CAS.

## Communicate Your Understanding

**C1** Jan calculated the  $x$ -intercepts of a quadratic relation as

$$x = \frac{-2 \pm \sqrt{5}}{8}.$$

- What are the individual  $x$ -intercepts?
- Describe how to use the  $x$ -intercepts to find the equation of the axis of symmetry.

**C2** After using the quadratic formula, explain how you would know if a quadratic equation has

- two real roots
- one real root
- no real roots

## Practise

For help with questions 1 and 2, see Example 1.

1. Use the quadratic formula to solve each equation. Express answers as exact roots.

- $7x^2 + 24x + 9 = 0$
- $2x^2 + 4x - 7 = 0$
- $4x^2 - 12x + 9 = 0$
- $2x^2 - 7x = -4$
- $3x^2 + 5x = 1$
- $16x^2 + 24x = -9$

2. **Use Technology** Use the quadratic formula to solve. Express your answers as exact roots and as approximate roots, rounded to the nearest hundredth. Verify graphically with technology.

- $3x^2 + 14x + 5 = 0$
- $8x^2 + 12x + 1 = 0$
- $4x^2 - 7x - 1 = 0$
- $10x^2 - 45x - 7 = 0$
- $-5x^2 + 16x - 2 = 0$
- $-6x^2 + 17x + 5 = 0$

For help with questions 3 to 5, see Examples 2 and 3.

3. Find the  $x$ -intercepts, the vertex, and the equation of the axis of symmetry of each quadratic relation. Then, sketch the parabola.

- $y = 5x^2 - 14x - 3$
- $y = 2x^2 - 5x - 12$
- $y = x^2 + 10x + 25$
- $y = 9x^2 - 24x + 16$
- $y = x^2 - 2x + 3$
- $y = -x^2 - 3x - 3$

4. For each quadratic relation, state the coordinates of the vertex and the direction of opening. Then, determine how many  $x$ -intercepts the relation has.

- $y = (x - 3)^2 + 2$
- $y = -(x - 2)^2 + 4$
- $y = 2(x + 4)^2 - 5$
- $y = -2(x + 3)^2 - 1$
- $y = (x - 5)^2$

5. Verify the number of  $x$ -intercepts for each relation in question 4 by using the quadratic formula to find the  $x$ -intercepts.

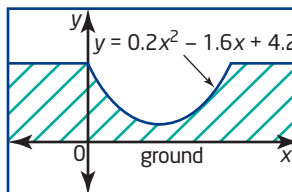
## Connect and Apply

For help with question 6, see Example 4.

6. The path of a soccer ball after it is kicked from a height of 0.5 m above the ground is given by the equation  $h = -0.1d^2 + d + 0.5$ , where  $h$  is the height, in metres, above the ground and  $d$  is the horizontal distance, in metres.
- How far has the soccer ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?
  - Find the horizontal distance when the soccer ball is at a height of 2.6 m above the ground.
7. A ball is thrown upward at an initial velocity of 8.4 m/s, from a height of 1.5 m above the ground. The height of the ball, in metres, above the ground, after  $t$  seconds, is modelled by the equation  $h = -4.9t^2 + 8.4t + 1.5$ .
- After how many seconds does the ball land on the ground? Round your answer to the nearest tenth of a second.
  - What is the maximum height, to the nearest metre, that the ball reaches?
8. Write an equation of a parabola, in the form  $y = a(x - h)^2 + k$ , satisfying each description. Then, write each relation in the form  $y = ax^2 + bx + c$ . Use graphing technology or the quadratic formula to verify that your equation satisfies the description.
- two  $x$ -intercepts
  - one  $x$ -intercept
  - no  $x$ -intercept
9. Solve. Round answers to the nearest hundredth, where necessary.
- $7x^2 - 12x = 9$
  - $4x^2 = 12 - 13x$
  - $4x^2 = 2.8x + 4.8$
  - $x(3x - 8) = -1$
  - $(x - 3)^2 = -2(x + 3)$
  - $(x + 3)^2 = (2x + 5)(2x - 5)$
10. The shape of the Humber River pedestrian bridge in Toronto can be modelled by the equation  $y = -0.0044x^2 + 21.3$ . All measurements are in metres. Determine the length of the bridge and the maximum height above the ground, to the nearest tenth of a metre.



11. A toy rocket is launched from a 3-m platform, at 8.1 m/s. The height of the rocket is modelled by the equation  $h = -4.9t^2 + 8.1t + 3$ , where  $h$  is the height, in metres, above the ground and  $t$  is the time, in seconds.
- After how many seconds will the rocket rise to a height of 6 m above the ground? Round your answer to the nearest hundredth.
  - When does the rocket fall again to a height of 6 m above the ground?
  - Use your answers from parts a) and b) to determine when the rocket reached its maximum height above the ground.
12. **Chapter Problem** The platforms on the ends of the half-pipe are at the same height.



- How wide is the half-pipe?
- How far would a skater have travelled horizontally after a drop of 2 m? Round to the nearest hundredth of a metre.

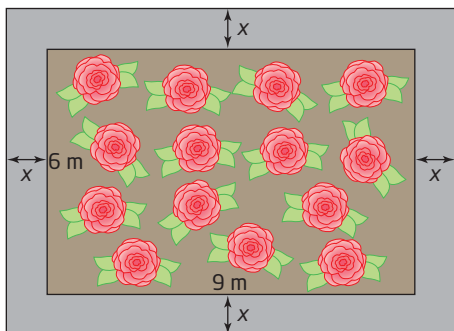
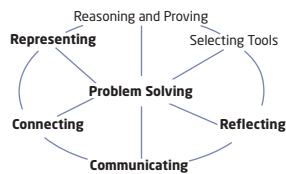
13. A shopping mall entrance contains a parabolic arch, modelled by the equation  $h = -0.5(d - 8)^2 + 32$ , where  $h$  is the height, in metres, above the floor and  $d$  is the distance, in metres, from one end of the arch. How wide is the arch at its base?

14. The fuel flowing to the engine of a small aircraft can be modelled by a quadratic equation over a limited range of speeds using the relation  $f = 0.0048v^2 - 0.96v + 64$ , where  $f$  represents the flow of fuel, in litres per hour, and  $v$  represents speed, in kilometres per hour.

- Show that this quadratic relation has no  $v$ -intercepts.
- Determine the speed that minimizes fuel flow.

### Achievement Check

15. The parks department is planning a new flower bed outside city hall. It will be rectangular with dimensions 9 m by 6 m. The flower bed will be surrounded by a path of constant width with the same area as the flower bed. Calculate the perimeter of the outside of the path.



- Set up a quadratic equation to model the question.
- Use the quadratic formula to solve the problem.
- Use Technology** Check your solution using a CAS or a graphing calculator.

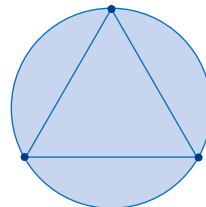
### Extend

16. Find a quadratic equation with each pair of roots.

a)  $x = \frac{14}{4} ; \sqrt{140}$

b)  $x = \frac{-11}{6} ; \sqrt{145}$

17. Points are drawn on a circle.



- If there are three points, how many line segments can be drawn joining any two points?
- What if there are four points? five points? six points?
- If there are  $n$  points, how many line segments can be drawn joining any two points?
- How many points are needed in order to have at least 1000 line segments?

18. When  $b^2 - 4ac = 0$ , there is only one real root. Give an example showing why this is so.

19. **Math Contest** Find the points of intersection of  $x^2 + y^2 = 16$  and  $y = x^2 - 9$ , and sketch a graph.

20. **Math Contest** What are the different possibilities for the number of points of intersection for a circle and a parabola? For each case, give an equation for the circle and for the parabola.

21. **Math Contest** Show that the roots of  $x = 1 + \frac{1}{x}$  are negative reciprocals. The positive root of  $x = 1 + \frac{1}{x}$  is called the *golden ratio*.

**22. Math Contest** A continued fraction is a fraction of the form  $a + \frac{1}{b + \frac{1}{c + \frac{1}{d + p}}}$ .

a) Evaluate the continued fraction

$$2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}$$

b) Evaluate the infinite continued

$$\text{fraction } 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + p}}}$$

**23. Math Contest** In 1202, Leonardo of Pisa (1175–1250), better known as Fibonacci, published a book called *Liber Abaci*. In it, he showed a sequence of numbers now called the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, ...

a) Find the next three terms and a formation rule for this sequence.

b) Examine the ratios of successive terms of the Fibonacci sequence

$$\frac{(n + 1)\text{th term}}{n\text{th term}}$$

Conjecture a possible value for the ratio  $\frac{1000\text{th term}}{999\text{th term}}$ .

**24. Math Contest** Solve the system of equations.

$$a - b + c = 0$$

$$a - c + d = 1$$

$$a + 2b + 2d = 0$$

$$b - d = 3$$

## Making Connections

A mind map can provide a mental picture of how many of the concepts you have learned about quadratics in Chapters 4, 5, and 6 are connected. One is started here. Make a larger version for yourself. Complete the shaded parts and add any other details that help you.

### Quadratic Relations

**A**  $y = ax^2 + bx + c$

$a > 0$  means

$a < 0$  means

$c$  is the

**B**  $y = a(x - h)^2 + k$

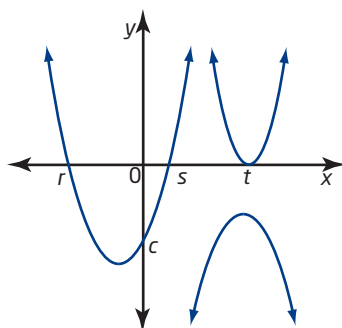
$a$  means the same as in form **A**

$(h, k)$  is the

**C**  $y = a(x - r)(x - s)$

$a$  means the same as in form **A**

$r$  and  $s$  are the



### Quadratic Equations

$$ax^2 + bx + c = 0$$

If the quadratic expression factors:

- $a(x - r)(x - s) = 0$

2 zeros are at

- $a(x - t)^2 = 0$  perfect square

1 zero at  $t$

- $a(x - p)(x + p) = 0$  difference of squares

roots are

axis of symmetry is

If no easy factors are found:

- complete the square to get form **B**

- use the quadratic formula

if no real roots exist, then parabola