

5.4 Solving from Vertex Form

Recall: Vertex Form

$$y = a(x - h)^2 + k$$

- vertex is (h, k)
- if $a > 0$, opens UP
- if $a < 0$, opens DOWN

1. Determine the number of x-intercepts, then determine their value.

a) $y = (x+2)^2 - 9$
 $V(-2, -9)$
 UP!
 $\therefore 2$ zeroes!
 $y=0$ at x-axis
 $0 = (x+2)^2 - 9$
 $9 = (x+2)^2$
 $\pm\sqrt{9} = \sqrt{(x+2)^2}$
 $\pm 3 = x+2$
 $\pm 3 - 2 = x$
 $\downarrow \quad \searrow$
 $x = +3 - 2 \quad x = -3 - 2$
 $= 1 \quad = -5$

Vertex?
 Opens up or
 down?

b) $y = -7(x-5)^2$
 $V(5, 0)$ DOWN!
 \therefore One zero
 $y=0$
 $0 = -7(x-5)^2$
 $0 = (x-5)^2$
 $\pm\sqrt{0} = x-5$
 $0 = x-5$
 $5 = x$

c) $y = -2(x+7)^2 - 8$
 $V(-7, -8)$ DOWN!
 \therefore NO zeroes!
 (Just to show you...)
 $0 = -2(x+7)^2 - 8$
 $8 = -2(x+7)^2$
 $-4 = (x+7)^2$
 $\pm\sqrt{-4} = x+7$
 Can't proceed

d) $y = -3(x-1)^2 + 2$
 $V(1, 2)$ DOWN
 $\therefore 2$ zeroes
 $y=0$
 $0 = -3(x-1)^2 + 2$
 $-2 = -3(x-1)^2$
 $\frac{2}{3} = (x-1)^2$
 $\pm\sqrt{\frac{2}{3}} = x-1$
 $\pm\sqrt{\frac{2}{3}} + 1 = x$
 $x = \sqrt{\frac{2}{3}} + 1$ ← EXACT → $x = -\sqrt{\frac{2}{3}} + 1$
 $= 1.8 \quad = 0.2$
 ← APPROX. →

2. Solve.

$$\begin{aligned} \text{a) } 0 &= 3(x-5)^2 - 12 \\ &V(5, -12) \text{ UP} \\ &\therefore 2 \text{ zeroes} \\ 0 &= 3(x-5)^2 - 12 \\ 12 &= 3(x-5)^2 \\ 4 &= (x-5)^2 \\ \pm\sqrt{4} &= x-5 \\ \pm 2 + 5 &= x \\ \downarrow & \quad \swarrow \\ 2+5 &= x & -2+5 &= x \\ 7 &= x & 3 &= x \end{aligned}$$

$$\begin{aligned} \text{b) } -2(x+7)^2 &= -5 \\ -2(x+7)^2 &= -5 & \begin{cases} -2(x+7)^2 + 5 = 0 \\ V(-7, 5) \\ \text{Down} \end{cases} \\ (x+7)^2 &= \frac{5}{2} \\ x+7 &= \pm\sqrt{\frac{5}{2}} \\ x &= \pm\sqrt{\frac{5}{2}} - 7 \\ &\swarrow \quad \searrow \\ x &= \sqrt{\frac{5}{2}} - 7 & x &= -\sqrt{\frac{5}{2}} - 7 \\ &= -5.4 & &= -8.6 \end{aligned}$$

$$\begin{aligned} \text{c) } 4(x-1)^2 + 6 &= 0 \\ &V(1, 6) \text{ UP!} \\ &\therefore \text{NO zeroes} \end{aligned}$$

$$\begin{aligned} \text{d) } 4(x-3)^2 &= 25 \\ &V(3, -25) \text{ UP} \\ &\therefore 2 \text{ zeroes} \\ 4(x-3)^2 &= 25 \\ (x-3)^2 &= \frac{25}{4} \\ x-3 &= \pm\sqrt{\frac{25}{4}} \\ x-3 &= \pm\frac{\sqrt{25}}{\sqrt{4}} \\ x-3 &= \pm\frac{5}{2} \\ x &= \pm\frac{5}{2} + 3 \\ &\swarrow \quad \searrow \\ x &= \frac{5}{2} + 3 & x &= -\frac{5}{2} + 3 \\ &= \frac{5}{2} + \frac{6}{2} & &= -\frac{5}{2} + \frac{6}{2} \\ &= \frac{11}{2} & &= \frac{1}{2} \\ &= 5.5 & &= 0.5 \end{aligned}$$

Practice!

Set 1: Handout 5.4 #1, 3, 5, 6, 8, 10

Set 2: Handout 5.4 #7, 8, 9, 10, 11, 12