5.5A Completing the Square

Ex. 1 Determine the value of the missing number so each trinomial is a perfect square, then factor it.

a) $x^2 - 6x + m$

$(x-3)^2$ $m = 9$

	x^2	$-3x$
-3	$-3x$	9

(−3)

b) $x^2 + 20x + m$

$(x+10)^2$ $m = 100$

x^2	$10x$
$10x$	100

c) $x^2 - 10x + m$

$(x-5)^2$ $m = 25$

x^2	$-5x$
$-5x$	25

d) $x^2 + 30x + m$

$(x+15)^2$ $m = 225$

$\left(\frac{+30}{2}\right)^2 = 225$

The process of completing the square allows you to change a quadratic equation from standard form to vertex form.

standard form $y = x^2 + 4x + 3$ ⟷ vertex form $y = (x+2)^2 - 1$

mulitply (←)

complete the square (→)

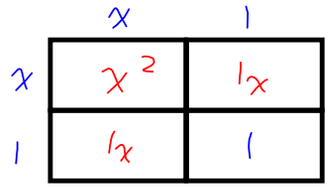
Ex. 2 Complete the square for the function $y = x^2 + 2x + 7$.

$y = x^2 + 2x + 7$

$\{y = x^2 + 2x + 1\} \quad -1 + 7$

$= (x+1)^2 + 6$

Steps:
- place the x^2 terms in the upper left box
- place the x terms in the upper right and lower left boxes (equal # in each spot)
- place the constant outside the box
- add a new constant in the lower right box to make a perfect square.
- use the zero principle to subtract what you just added (leave outside the box)
- write the expression in vertex form



+7 -1

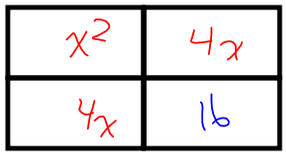
$= (x+1)^2 + 6$

Ex. 3 Rewrite each equation in vertex form by using the box to complete the square.

a) $y = x^2 + 8x - 3$

$= (x+4)^2 - 19$

Steps:
- place the x^2 terms in the upper left box
- place the x terms in the upper right and lower left boxes (equal # in each spot)
- place the constant outside the box
- add a new constant in the lower right box to make a perfect square.
- use the zero principle to subtract what you just added (leave outside the box)
- write the expression in vertex form

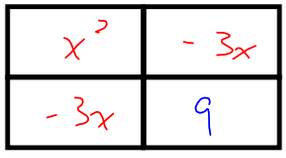


-3

-16

b) $y = x^2 - 6x + 2$

$= (x-3)^2 - 7$



+2

-9

Ex. 4 Rewrite $y = x^2 - 14x - 5$ in vertex form by algebraically completing the square.

Box

$y = x^2 - 14x - 5$

$= (x-7)^2 - 54$

x^2	$-7x$
$-7x$	49

-5

-49

Algebraically

$y = x^2 - 14x - 5$

$= \underbrace{x^2 - 14x + 49}_{\left(\frac{14}{2}\right)^2 = 49} - 49 - 5$

$= (x-7)^2 - 54$

Ex. 5 Rewrite each of the following in vertex form by completing the square with the box, then algebraically.

a) $y = x^2 - 10x - 4$

Box

$= (x-5)^2 - 29$

x^2	$-5x$
$-5x$	25

-4

-25

Algebraically

$y = x^2 - 10x - 4$

$= \underbrace{x^2 - 10x + 25}_{} - 25 - 4$

$= (x-5)^2 - 29$

b) $y = x^2 + 12x - 5$

Box

Algebraically

$y = \underbrace{x^2 + 12x + 36}_{} - 36 - 5$

$= (x+6)^2 - 41$