5.6 Solving by Completing the Square

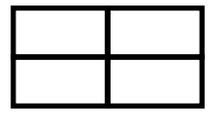
Ex. 1 Solve. $x^2 + 2x - 15 = 0$

Does it factor?

Yes!

$(x-5)(x+3) = 0$

$x = 5 \quad x = -3$



Ex. 2 Solve. $x^2 + 2x - 5 = 0$

Does it factor? NO!

What other form can we solve from?

$(x+1)^2 - 6 = 0$

$(x+1)^2 = 6$

$x+1 = \pm\sqrt{6}$

$x = \pm\sqrt{6} - 1$

$x = \sqrt{6} - 1 \quad x = -\sqrt{6} - 1$

$\doteq 1.4 \quad \doteq -3.4$

	x	$+1$
x	x^2	x
$+1$	x	1

-5

-1

Ex. 3 Solve. $x^2 + 4x + 7 = 0$

Does it factor? NO!

$x^2 + 4x + 4 - 4 + 7 = 0$

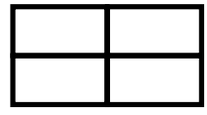
$(x+2)^2 + 3 = 0$

$(x+2)^2 = -3$

$x+2 = \pm\sqrt{-3}$

Can't take root of negative.

∴ No roots!



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Ex. 4 Solve each of the following.

a) $x^2 - 6x + 4 = 0$

$x^2 - 6x + 9 - 9 + 4 = 0$

$(x-3)^2 - 5 = 0$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$x = \pm\sqrt{5} + 3$

$x = \sqrt{5} + 3 = 5.2$

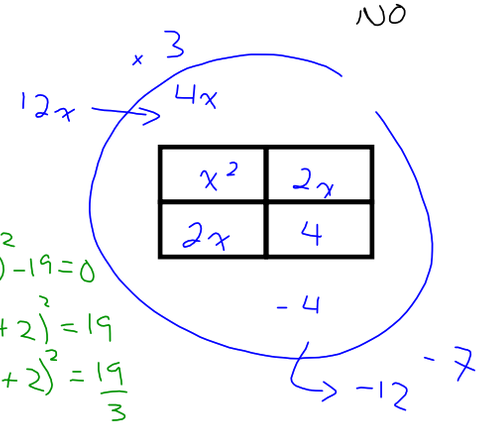
$x = -\sqrt{5} + 3 = 0.8$

b) $3x^2 + 12x - 7 = 0$

$3(x+2)^2 - 19 = 0$

Does it factor?

NO



$3(x+2)^2 - 19 = 0$

$3(x+2)^2 = 19$

$(x+2)^2 = \frac{19}{3}$

$x + 2 = \pm\sqrt{\frac{19}{3}}$

$x = \pm\sqrt{\frac{19}{3}} - 2$

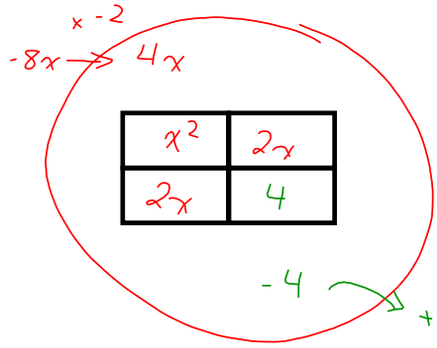
$x \doteq 0.5$ $\quad x \doteq -4.5$

c) $-2x^2 - 8x - 14 = 0$

$-2(x^2 + 4x + 4 - 4) - 14 = 0$

$-2(x^2 + 4x + 4) + 8 - 14 = 0$

$-2(x+2)^2 - 6 = 0$



$-2(x+2)^2 - 6 = 0$

$-2(x+2)^2 = 6$

$(x+2)^2 = -3$

$x + 2 = \pm\sqrt{-3}$

no roots!

d) $4x^2 - 8x - 5 = 0$

$4(x-1)^2 - 9 = 0$

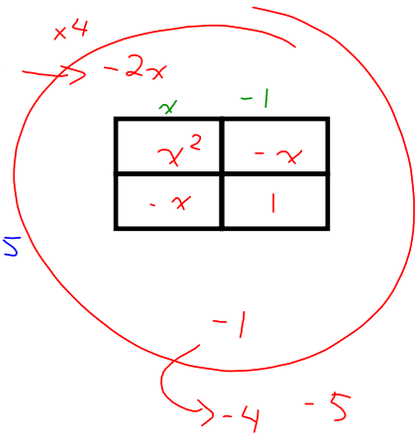
$4(x-1)^2 = 9$

$(x-1)^2 = \frac{9}{4}$

$x - 1 = \pm\sqrt{\frac{9}{4}}$

$x = \pm\sqrt{\frac{9}{4}} + 1$

$x = 2.5$ $\quad x = -0.5$



Ex. 5 Convert to vertex form, then solve to find the zeros.
Graph using the vertex and zeros.

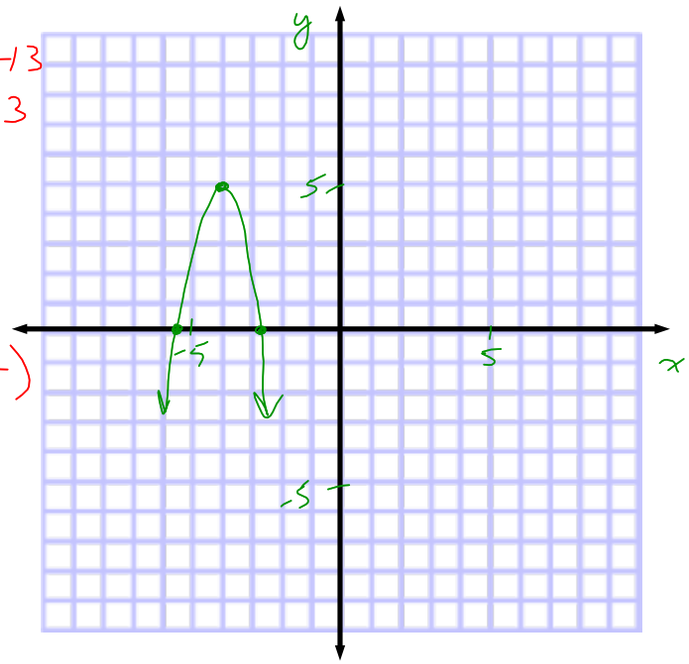
$y = -3x^2 - 24x - 43$

$= -3(x^2 + 8x + 16 - 16) - 43$

$= -3(x^2 + 8x + 16) + 48 - 43$

$= -3(x+4)^2 + 5$

Vertex
$(-4, 5)$



Solve

$0 = -3(x+4)^2 + 5$

$-5 = -3(x+4)^2$

$\frac{5}{3} = (x+4)^2$

$\pm\sqrt{\frac{5}{3}} = x + 4$

$\pm\sqrt{\frac{5}{3}} - 4 = x$

$x \doteq -2.7$ $\quad$ $x \doteq -5.3$