5.7 The Quadratic Formula

A. Finding Zeroes from VERTEX Form:

Ex. 1 Find the zeroes of $y = 2(x - 1)^2 - 18$.

$0 = 2(x-1)^2 - 18$

$18 = 2(x-1)^2$

$9 = (x-1)^2$

$\pm 3 = x - 1$

$\pm 3 + 1 = x$

$x = 3 + 1 = 4$

$x = -3 + 1 = -2$

B. Finding Zeroes from Standard Form (if it factors):

Ex. 2 Find the roots/zeroes/x-int.

Factor?

$y = 2x^2 - 11x + 5$

$0 = (2x-1)(x-5)$

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$x - 5 = 0$

$x = 5$

	x	-5
$2x$	$2x^2$	$-10x$
-1	$-1x$	5

M 10

A -11

N $-10, -1$

C. Finding Zeroes from Standard form (doesn't factor):

Ex. 3 Find the roots/zeroes/x-int.

Need to complete the square! THEN solve

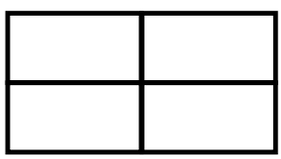
$y = 2x^2 + 9x + 6$

$y = 2(x^2 + \frac{9}{2}x + \frac{81}{16} - \frac{81}{16}) + 6$

$(\frac{9}{2} \div 2)^2 = \frac{9}{4}$

$= \ldots\ldots$

gross.....



Is there a shorter way?

Please.....

C. Finding Zeroes from Standard form (doesn't factor):

Strategy:
- put in vertex form first
- solve from vertex form

Solve for x by completing the square
$2x^2 - 11x + 5 = 0$.

$2(x^2 - \frac{11}{2}x \quad) + 5 = 0$ ($\frac{11}{2} \to (\frac{11}{4})^2$)

$2(x^2 - \frac{11}{2}x + \frac{121}{16} - \frac{121}{16}) + 5 = 0$

$2(x^2 - \frac{11}{2}x + \frac{121}{16}) - \frac{121}{8} + 5 = 0$

$2(x - \frac{11}{4})^2 - \frac{121}{8} + \frac{40}{8} = 0$

$2(x - \frac{11}{4})^2 - \frac{81}{8} = 0$

$2(x - \frac{11}{4})^2 = \frac{81}{8}$

$(x - \frac{11}{4})^2 = \frac{81}{16}$

$x - \frac{11}{4} = \pm\sqrt{\frac{81}{16}}$

$x - \frac{11}{4} = \pm\frac{9}{4}$

$x = \pm\frac{9}{4} + \frac{11}{4}$

$= \frac{11 \pm 9}{4}$

Develop a general formula:

Solve for x by completing the square.
$ax^2 + bx + c = 0$

$a(x^2 + \frac{b}{a}x \quad) + c = 0$ ($\frac{b}{a} \to (\frac{b}{2a})^2$)

$a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}) + c = 0$

$a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) - \frac{b^2}{4a} + c = 0$

$a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + \frac{4ac}{4a} = 0$

NOW SOLVE

$a(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a}$

$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$x = \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

To find the roots when the equation is in standard form and DOES NOT factor use:

The Quadratic Formula:
For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is called the DISCRIMINANT

Ex. 3 Solve each of the following using the quadratic formula:

a) $2x^2-5x-1 = 0$

$a = 2$, $b = -5$, $c = -1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-5) \pm \sqrt{(-5)^2-4(2)(-1)}}{2(2)}$

$= \frac{5 \pm \sqrt{33}}{4}$

$= \frac{5+\sqrt{33}}{4}$ $= \frac{5-\sqrt{33}}{4}$

$\doteq 2.7$ $\doteq -0.2$

b) $x^2 - 30x + 225 = 0$

$a = 1$, $b = -30$, $c = 225$

$x = \frac{-(-30) \pm \sqrt{(-30)^2-4(1)(225)}}{2(1)}$

$= \frac{30 \pm \sqrt{0}}{2}$

$= \frac{30}{2}$

$= 15$

Only one root!

c) $3x^2+2x = -15$

$3x^2+2x+15 = 0$

$a = 3$, $b = 2$, $c = 15$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2 \pm \sqrt{2^2-4(3)(15)}}{2(3)}$

$= \frac{-2 \pm \sqrt{-176}}{6}$

Can't take square root!

∴ No "real" roots

No zeros

d) $3x(2x + 1) = -2(x - 4)$

$6x^2+3x = -2x+8$

$6x^2+5x-8 = 0$

$a = 6$, $b = 5$, $c = -8$

$x = \frac{-5 \pm \sqrt{5^2-4(6)(-8)}}{2(6)}$

$= \frac{-5 \pm \sqrt{217}}{12}$

$x = \frac{-5+\sqrt{217}}{12}$ $x = \frac{-5-\sqrt{217}}{12}$

$\doteq 0.8$ $\doteq -1.6$