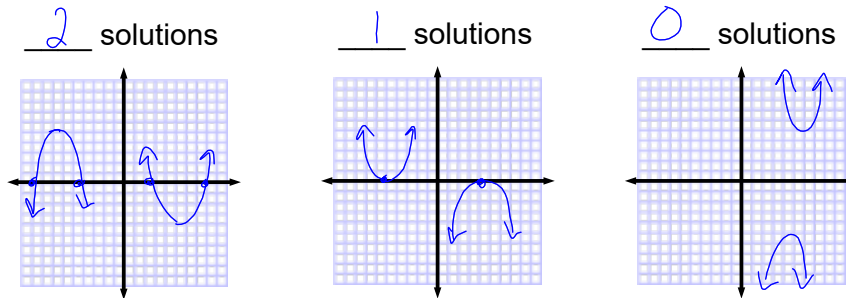


## 5.8 The Discriminant

How many zeros/solutions/x-int/roots does a quadratic have?

Sketch the possibilities.



### A. # of Zeros from VERTEX Form

Ex. 1 State the vertex, direction of opening, and # of zeros.

<p>a) <math>y = -2(x+3)^2 + 5</math></p> <p>V (-3, 5)</p> <p>DOWN</p> <p> ∴ Two Zeros</p>	<p>b) <math>y = 5(x - 4)^2</math></p> <p>V (4, 0)</p> <p>UP</p> <p> ∴ One Zero</p>	<p>c) <math>y = 3(x - 7)^2 + 2</math></p> <p>V (7, 2)</p> <p>UP</p> <p> ∴ No Zeros</p>
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### B. # of Zeros from Standard Form

Ex. 2 Determine the number of zeros by solving using the quadratic formula.

<p>a) <math>y = 5x^2 - 3x + 9</math></p> <p><math>a=5 \quad b=-3 \quad c=9</math></p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(9)}}{2(5)}$ $= \frac{3 \pm \sqrt{-171}}{10}$ <p>∴ NO ZEROS</p>	<p>b) <math>y = 3x^2 + 5x - 4</math></p> <p><math>a=3 \quad b=5 \quad c=-4</math></p> $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)}$ $= \frac{-5 \pm \sqrt{73}}{6}$ <p>∴ Two Zeros</p>	<p>c) <math>y = 9x^2 - 12x + 4</math></p> <p><math>a=9 \quad b=-12 \quad c=4</math></p> $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$ $= \frac{12 \pm \sqrt{0}}{18}$ <p>∴ One Zero</p>
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Can you tell before you finish solving how many solutions there will be? How?

The Quadratic Formula:

$$\text{For } ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$  is called the **DISCRIMINANT**

The value of the discriminant,  $D = b^2 - 4ac$  determines whether a quadratic equation has 0, 1, or 2 solutions/zeros/roots.

Ex. 5 Determine the discriminant,  $D$ , then state the number of roots/solutions/zeros).

a)  $0 = 3x^2 + 7x + 9$

$a=3 \quad b=7 \quad c=9$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 7^2 - 4(3)(9) \\ &= -59 \end{aligned}$$

$D < 0$   
 $\therefore$  NO Zeros!

b)  $0 = 5x^2 - 8x - 3$

$a=5 \quad b=-8 \quad c=-3$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-8)^2 - 4(5)(-3) \\ &= 124 \end{aligned}$$

$D > 0$   
 $\therefore$  2 roots

c)  $0 = 4x^2 - 20x + 25$

$a=4 \quad b=-20 \quad c=25$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-20)^2 - 4(4)(25) \\ &= 0 \end{aligned}$$

$D = 0$   
 $\therefore$  1 root!

d)  $0 = 10x^2 + x - 3$

$a=10 \quad b=1 \quad c=-3$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 1^2 - 4(10)(-3) \\ &= 121 \end{aligned}$$

$D > 0$   
 $\therefore$  2 roots!

**Summary:**

- If  $D > 0$ , then the quadratic equation has 2 roots.
- If  $D = 0$ , then the quadratic equation has 1 root.
- If  $D < 0$ , then the quadratic equation has 0 roots.