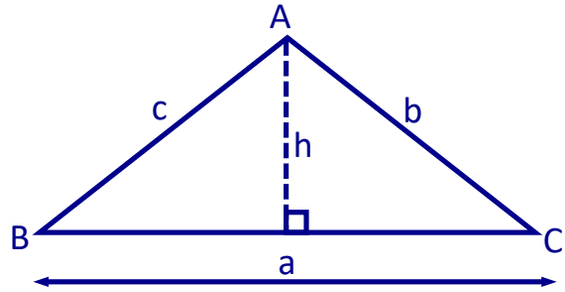
Lesson 4.4A Problems in Two Dimensions Day 1

Development of the Sine Law:

Consider ΔABC (no 90° angle).

Construct an altitude from A.

There are now 2 right triangles.



STEPS:

1. Write equations for sin B and sin C. ⇨ $\sin B = \frac{h}{c}$ $\sin C = \frac{h}{b}$

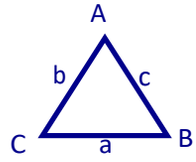
2. Solve each equation for h. ⇨ $c\sin B = h$ $b\sin C = h$

3. Since both equations = h, they must equal each other. ⇨ $\therefore\ c\sin B = b\sin C$

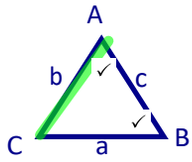
4. Divide both sides by b and c. ⇨ $\frac{\sin B}{b} = \frac{\sin C}{c}$

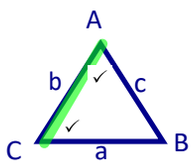
The Sine Law

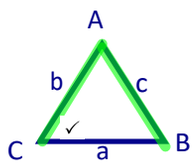
In ΔABC,



$$\frac{a}{sinA} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad or \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

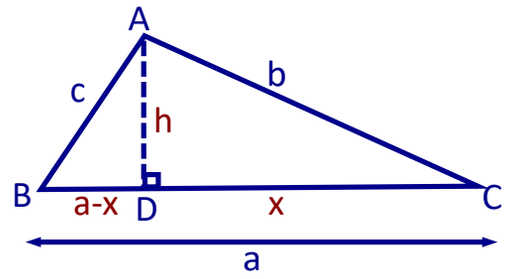






Development of the Cosine Law:

- consider ΔABC (no 90°)
- construct an altitude from A
- notice that there are now 2 right triangles



In ΔADC:

- Write the Pythagorean theorem

$$b^2 = x^2 + h^2$$

$$\therefore h^2 = b^2 - x^2$$

- Write the cosine ratio for C

$$\cos C = \frac{x}{b}$$

$$x = b \cdot \cos C$$

In ΔABD:

- Write the Pythagorean theorem
- Expand and simplify
- Substitute from ΔADC

$$c^2 = h^2 + (a - x)^2$$

$$c^2 = h^2 + a^2 - 2ax + x^2$$

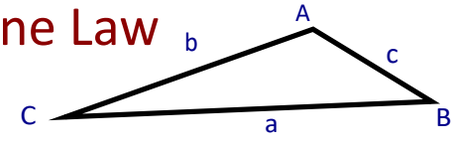
$$c^2 = (b^2 - x^2) + a^2 - 2ax + x^2$$

$$c^2 = b^2 + a^2 - 2ax$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

The Cosine Law

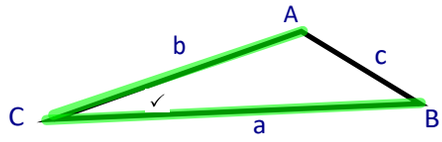
In ΔABC,

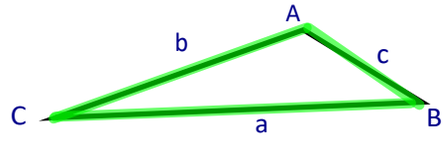


$c^2 = a^2 + b^2 - 2ab \cos C$ rearrange --> $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

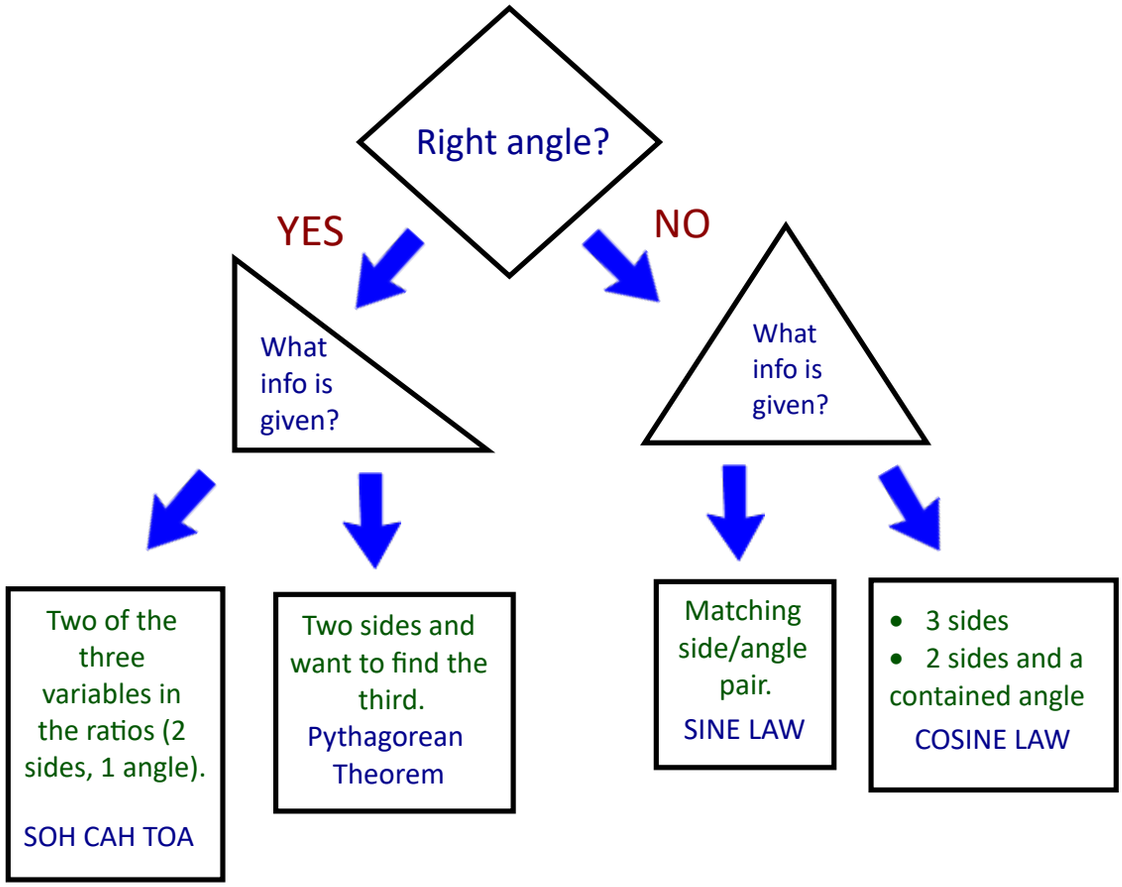
(Used when finding a side).

(Used when finding an angle).

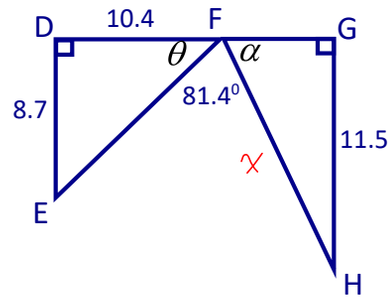




Which tool should I use?



Ex. 1 Find the length of FH.



Steps?/Process?

③ ② ① (bottom to top)

- Use PTR to find θ
- Total of θ + α + 81.4 = 180
- Use ∠ & $\sin\alpha = \frac{O}{H}$

① Use tan to find θ

$\tan\theta = \frac{8.7}{10.4}$

$\theta \doteq 39.9°$

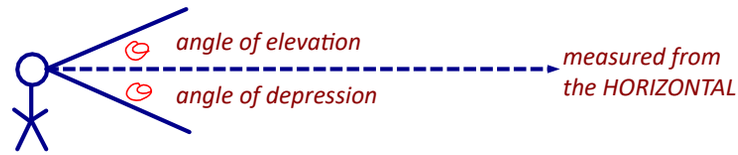
② $39.9 + 81.4 + \alpha = 180$

$\alpha = 58.7°$

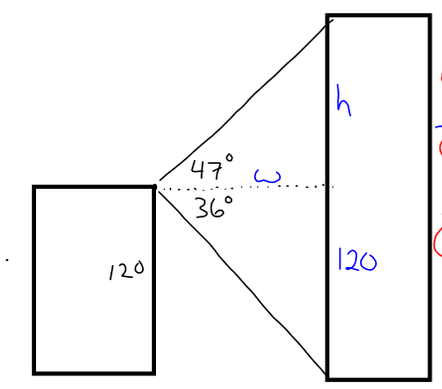
③ $\sin 58.7° = \frac{11.5}{x}$

$x \doteq 13.5$

∴ The length of FH is 13.5



Ex. 2 From the top of a 120 m tall building, the angle of elevation to the top of another building is 47° and the angle of depression to the bottom of the same building is 36°. How high is the second building?



Plan

- Use tan to find w ①
- Use tan to find h ②
- Add h + 120 ③

① w, 36°, 120

$\tan 36° = \frac{120}{w}$

$w \doteq 165.2$

② h, 47°, 165.2

$\tan 47° = \frac{h}{165.2}$

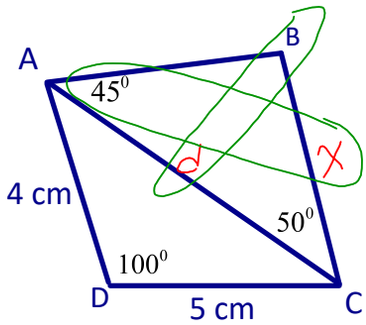
$h \doteq 177.2$

③ total height = 177.2 + 120

= 297.2

∴ The height of the second building is 297 m

Ex. 3 Find the length of BC to one decimal place.



1 - cosine law to find d
2 - sum of angles for B
3 - sine law for x

① $d^2 = 4^2 + 5^2 - 2(4)(5)\cos 100^\circ$
$d \doteq 6.9$

② $B = 180 - 45 - 50$
$= 85^\circ$

③ $\frac{x}{\sin 45} = \frac{6.9}{\sin 85}$

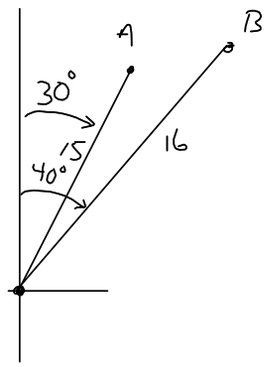
$x = 4.9$

∴ Side BC is 4.9 cm

Ex. 4 A radar screen at an air traffic control tower shows that airplane A is 15 km from the tower in a direction 30^0 east of north. A second airplane, airplane B, is 16 km from the tower in a direction 40^0 east of north.

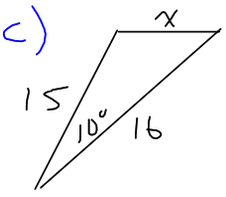
a) Sketch a diagram, labelling given values.
b) From the tower, what is the angle separating the aircraft?
c) How far apart are the two aircraft?

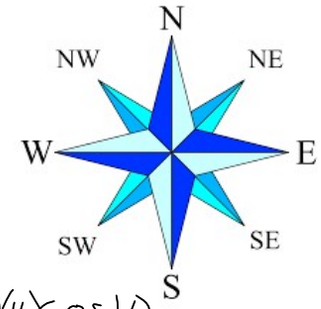
a)



b) 10°

c)





$x^2 = 15^2 + 16^2 - 2(15)(16)\cos 10$
$x \doteq 2.9$

∴ The plane are 2.9 km apart.

Homework
pg. 318 # 4,7
pg. 326 # 3, 4, 5
These challenges below!