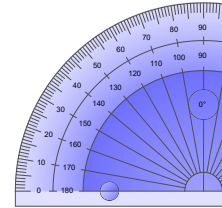
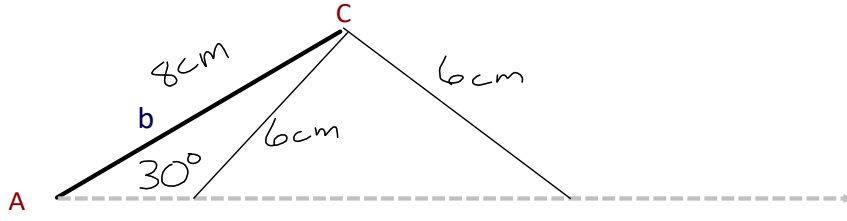
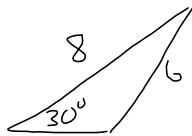


Lesson 4.4B: Sine Law - AMBIGUOUS Case

Draw triangle ΔABC , $a = 6$ cm, $b = 8$ cm, $A = 30^\circ$.



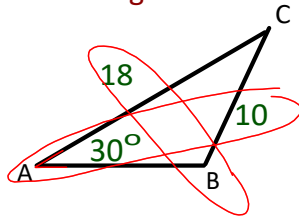
Which is it?



Must account for BOTH!

- When two sides and the non-included angle of a triangle are given, the triangle may not be unique. (SSA)
- You will have to determine if there is **no** solution, **one** solution or **two** possible solutions.

Ex. 1: Given that ΔABC has $\angle A = 30^\circ$, $a = 10$, and $b = 18$, find the value of $\angle B$ to the nearest degree.



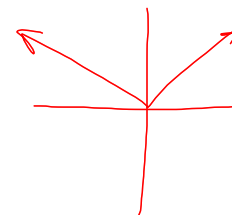
$$\frac{\sin B}{18} = \frac{\sin 30^\circ}{10}$$

$$\sin B = 0.9$$

$$\theta = 64^\circ$$

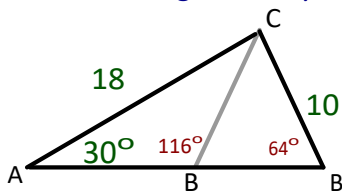
$$\frac{Q_1}{\theta} = 64^\circ$$

$$\frac{Q_2}{\theta} = 180 - 64 = 116^\circ$$



$$\therefore \angle B = 64^\circ, 116^\circ$$

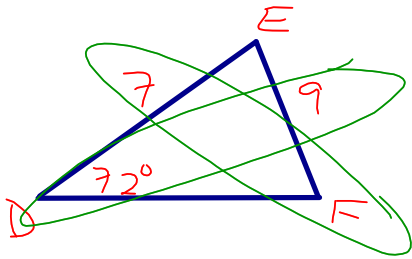
As we see algebraically, there are two possible answers to this question.



Therefore, it is very important to always *consider* both solutions (Q1 & Q2) when using Sine Law to solve a triangle given SSA.

Ex. 2: Determine the measures of all angles in the given triangles.

a) In $\triangle DEF$, $\angle D = 72^\circ$, $d = 9$ cm, $f = 7$ cm.



$$\frac{\sin F}{7} = \frac{\sin 72}{9}$$

$$\frac{Q_1}{\theta} = 48^\circ$$

$$\sin F = 7 \cdot \frac{\sin 72}{9}$$

$$\frac{Q_2}{\theta} = 180 - 48^\circ = 132^\circ$$

$$F = 48^\circ$$

SSA!
Need to account for ambiguous case

Triangle 1

$$\angle E = 180 - 72 - 48 = 60^\circ \quad \text{VALID}$$

Triangle 2

$$\angle E = 180 - 72 - 132 = -24^\circ \quad \text{INVALID}$$

\therefore Only 1 solution

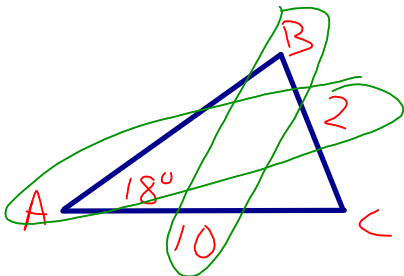
$$\angle E = 60^\circ$$

$$\angle F = 48^\circ$$

$$\angle D = 72^\circ$$

Pull

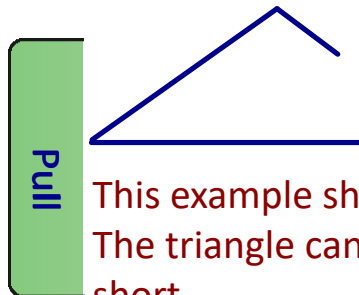
b) In $\triangle ABC$, $\angle A = 18^\circ$, $a = 2$ cm, $b = 10$ cm.



$$\frac{\sin B}{10} = \frac{\sin 18}{2}$$

ERROR no solutions!

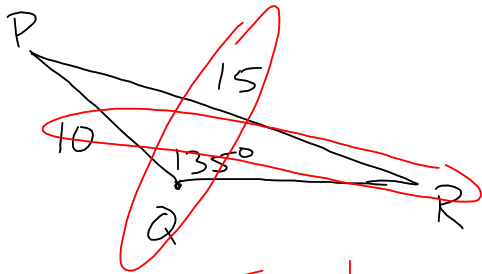
SSA!



Pull

This example shows the case of no solution. The triangle cannot be constructed as side "a" is too short.

c) In ΔPQR , $\angle Q = 135^\circ$, $q = 15$ cm, $r = 10$ cm.



$$\frac{\sin R}{10} = \frac{\sin 135}{15}$$

$$\angle R = 28^\circ$$

SSA!

$$\angle Q_1 = 28^\circ$$

$$\angle Q_2 = 180 - 28 = 152^\circ$$

Both valid? Triangle 1

$$\angle P = 180 - 135 - 28 = 17^\circ$$

\therefore Only 1 Solution

$$\angle P = 17^\circ, \angle R = 28^\circ$$

$$\angle Q = 135^\circ$$

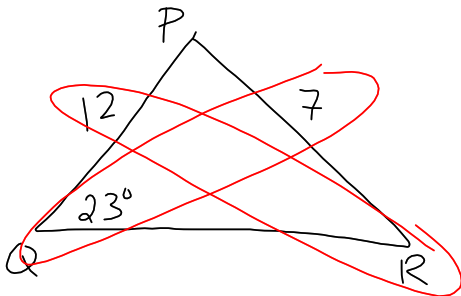
Triangle 2

$$\angle P = 180 - 152 - 135$$

INVALID

Pull

d) In ΔPQR , $\angle Q = 23^\circ$, $q = 7$ cm, $r = 12$ cm.



SSA!

$$\frac{\sin R}{12} = \frac{\sin 23}{7}$$

$$\angle R = 42^\circ$$

$$\angle Q_1 = 42^\circ$$

$$\angle Q_2 = 138^\circ$$

Triangle 1 VALID

$$\angle P = 180 - 23 - 42 = 115^\circ$$

$$\angle Q = 23^\circ$$

$$\angle R = 42^\circ$$

Triangle 2 VALID

$$\angle P = 180 - 23 - 138 = 19^\circ$$

$$\angle Q = 23^\circ$$

$$\angle R = 138^\circ$$

2 solutions

Pull

Homework
pg. 318 #2, 3, 4a, 5, 6

am•big•u•ous

doubtful, uncertain,
unclear in meaning