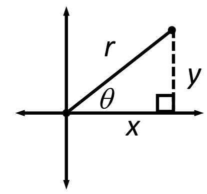
Lesson 4.6 A: Trig Identities (Day 1)

Recall the following definitions:

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$ $\quad x^2 + y^2 = r^2$



An identity is an equation that is always true, regardless of the value of the variable.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

1) The Quotient Identities

$$\frac{\sin\theta}{\cos\theta} = \frac{y}{r} \div \frac{x}{r}$$

$$= \frac{y}{\cancel{r}} \times \frac{\cancel{r}}{x}$$

$$= \frac{y}{x}$$

$$= \tan\theta$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta} \qquad \boxed{\frac{\cos\theta}{\sin\theta} = \cot\theta}$$

Note:

$$\boxed{\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta} \qquad \boxed{\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta}$$

2) The Pythagorean Identities

$\sin^2\theta + \cos^2\theta =$

$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{y^2 + x^2}{r^2} \quad \rightarrow r^2$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

This identity is often rearranged to give:

$$\boxed{\sin^2\theta = 1 - \cos^2\theta}$$

$$\boxed{\cos^2\theta = 1 - \sin^2\theta}$$

3) The Reciprocal Identities

$$\boxed{\sin\theta = \frac{1}{\csc\theta}} \quad \boxed{\cos\theta = \frac{1}{\sec\theta}} \quad \boxed{\tan\theta = \frac{1}{\cot\theta}}$$

$$\boxed{\csc\theta = \frac{1}{\sin\theta}} \quad \boxed{\sec\theta = \frac{1}{\cos\theta}} \quad \boxed{\cot\theta = \frac{1}{\tan\theta}}$$

These identities can also be squared.

<u>STEPS TO PROVING IDENTITIES</u>

1. <u>Separate</u> LS from RS.
2. Write both sides in terms of sin x and cos x.
3. To make LS = RS , try :
    - Factoring.
    - Simplifying.
    - Substitute any of the identities we just learned.
    - In some situations, multiply by the conjugate.

Examples: Prove each identity.

a) $\cos x \tan x = \sin x$

LS	RS
$= \cos x \tan x$	$= \sin x$
$= \cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}}$	
$= \sin x$	

$\therefore LS = RS$

$\therefore$ QED!

PULL

"Quod Erat Demonstrandum"

Which is what had to be proven.

b) $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = \dfrac{2}{\cos^2 x}$

LS	RS
$= \dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x}$ $\left\{ \dfrac{1}{1+a} + \dfrac{1}{1-a} \right.$	$= \dfrac{2}{\cos^2 x}$
$= \dfrac{1}{1+\sin x} \dfrac{(1-\sin x)}{(1-\sin x)} + \dfrac{1}{1-\sin x} \dfrac{(1+\sin x)}{(1+\sin x)}$	
$= \dfrac{1-\sin x + 1+\sin x}{(1+\sin x)(1-\sin x)}$	
$= \dfrac{2}{1-\sin^2 x}$ Pythagorean Identity	
$= \dfrac{2}{\cos^2 \theta}$	

$\therefore LS = RS$

$\therefore$ QED!

c) $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

LS

$$\begin{aligned}
&= \tan^2 x - \sin^2 x \\
&= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x \, \frac{\cos^2 x}{\cos^2 x} \\
&= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} \\
&= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}
\end{aligned}$$

Pythag. Identity

$$= \frac{\sin^2 x \cdot \sin^2 x}{\cos^2 x}$$

$$\begin{aligned}
&\frac{x^2}{y^2} - x^2 \\
&= \frac{x^2}{y^2} - x^2\left(\frac{y^2}{y^2}\right) \\
&= \frac{x^2 - x^2y^2}{y^2}
\end{aligned}$$

RS

$$\begin{aligned}
&= \sin^2 x \tan^2 x \\
&= \sin^2 x \cdot \frac{\sin^2 x}{\cos^2 x}
\end{aligned}$$

$\therefore LS = RS$

$\therefore$ QED!

d) $\dfrac{\cos x - \sin x - \cos^3 x}{\cos x} = \sin^2 x - \tan x$

LS

$$\begin{aligned}
&= \frac{\cos x - \sin x - \cos^3 x}{\cos x} \\
&= \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} - \frac{\cos^3 x}{\cos x} \\
&= 1 - \frac{\sin x}{\cos x} - \cos^2 x
\end{aligned}$$

$= \sin^2 x$

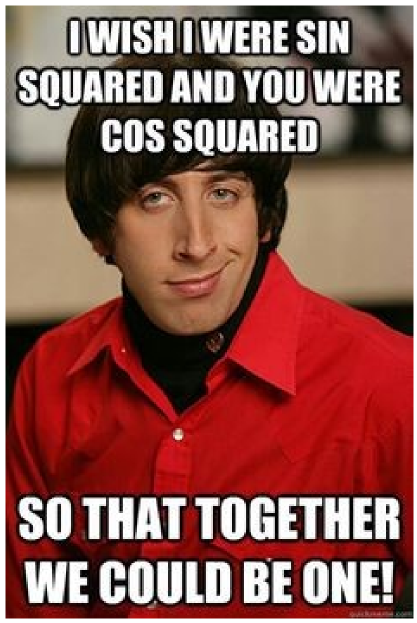
$$= \sin^2 x - \frac{\sin x}{\cos x}$$

RS

$$\begin{aligned}
&= \sin^2 x - \tan x \\
&= \sin^2 x - \frac{\sin x}{\cos x}
\end{aligned}$$

$\therefore LS = RS$

$\therefore$ QED!



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Identities Handout, Day 1 Questions