LS

$= \dfrac{1+2\sin\theta\cos\theta}{\sin\theta+\cos\theta}$

RS

$= \sin\theta + \cos\theta$ $\quad \dfrac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}$

$1 =$

$= \dfrac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\sin\theta+\cos\theta}$

$= \dfrac{1+2\sin\theta\cos\theta}{\sin\theta+\cos\theta}$

LS = RS

$\therefore$ QED

LS

$$= \frac{\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} \quad = 1$$

$$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{1}$$

$$= \sin^2\theta$$

RS

$$= \sin^2\theta$$

$\therefore LS = RS$

$\therefore$ QED

Lesson 4.6B - Trig Identities (Day 2)

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\csc^2\theta = \frac{1}{\sin^2\theta} \qquad \sec^2\theta = \frac{1}{\cos^2\theta} \qquad \cot^2\theta = \frac{1}{\tan^2\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \qquad \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta$$

Strategies

STEPS TO PROVING IDENTITIES

1. Separate LS from RS.
2. Write both sides in terms of sin x and cos x.
3. To make LS = RS , try :
   - Factoring.
   - Simplifying.
   - Substitute any of the identities we just learned.
   - In some situations, multiply by the conjugate.

Factoring

$1-\cos^2\theta$

$=(1-\cos\theta)(1+\cos\theta)$

$9-x^2$

$=(3-x)(3+x)$

Difference of Squares!

$\sin x-\sin^2 x$

$=\sin x(1-\sin x)$

$\sin^2\theta-2\sin\theta+1$

$=(\sin\theta-1)^2$

x^2-2x+1

$=(x-1)^2$

$\sin^2\theta-\cos^2\theta$

$=(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)$

x^2-y^2

$=(x-y)(x+y)$

Diff. of Squares

$\cos^2\theta-7\cos\theta+10$

$=(\cos\theta-5)(\cos\theta-2)$

M 10

A -7

N -5, -2

$6\sin^2\theta-\sin\theta-1$

$=(3\sin\theta+1)(2\sin\theta-1)$

M -6

A -1

N $\frac{2}{6}$ $\frac{-3}{6}$

$\frac{1}{3}$ $\frac{-1}{2}$

Multiplying by the conjugate

$\frac{1}{1-\cos x}$ — Conjugate? Same two terms with OPPOSITE SIGN

$=\frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}$

$=\frac{1+\cos x}{1-\cos^2 x}$

Examples - Prove the following identities.

a) $\dfrac{1}{\cot x} = \sin x \sec x$

LS	RS
$= \dfrac{1}{\cot x}$	$= \sin x \sec x$
$= \dfrac{\sin x}{\cos x}$	$= \sin x \dfrac{1}{\cos x}$
	$= \dfrac{\sin x}{\cos x}$

LS = RS

$\therefore$ QED

b) $\dfrac{1+\cot x}{\csc x} = \sin x + \cos x$

LS	RS
$= \dfrac{1+\cot x}{\csc x}$	$= \sin x + \cos x$
$= \dfrac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$	

$= \dfrac{\sin x + \cos x}{\sin x} \div \dfrac{1}{\sin x}$

$= \dfrac{\sin x + \cos x}{\cancel{\sin x}} \times \dfrac{\cancel{\sin x}}{1}$

$= \sin x + \cos x$

$\therefore$ LS = RS

$\therefore$ QED

c) $\dfrac{\cos\theta - 1}{1 - \sec\theta} = \dfrac{\cos\theta + 1}{1 + \sec\theta}$

LS	RS
$= \dfrac{\cos\theta - 1}{1 - \sec\theta} \cdot \dfrac{1 + \sec\theta}{1 + \sec\theta}$	$= \dfrac{\cos\theta + 1}{1 + \sec\theta} \cdot \dfrac{1 - \sec\theta}{1 - \sec\theta}$
$= \dfrac{\cos\theta + \cos\theta\sec\theta - 1 - \sec\theta}{1 - \sec^2\theta}$	$= \dfrac{\cos\theta - \cancel{\cos\theta\sec\theta}^{-1} + 1 - \sec\theta}{1 - \sec^2\theta}$
$= \dfrac{\cos\theta + \cancel{\cos\theta}^{1}\dfrac{1}{\cos\theta} - 1 - \sec\theta}{1 - \sec^2\theta}$	$= \dfrac{\cos\theta - \sec\theta}{1 - \sec^2\theta}$
$= \dfrac{\cos\theta - \sec\theta}{1 - \sec^2\theta}$	

$\therefore LS = RS$

$\therefore$ QED

d) $\tan\alpha\sin\alpha+\cos\alpha=\sec\alpha$

LS	RS
$=\tan x\sin x+\cos x$	$=\sec x$
$=\frac{\sin x}{\cos x}\cdot\sin x+\cos x$	$=\frac{1}{\cos x}$
$=\frac{\sin^2 x}{\cos x}+\frac{\cos^2 x}{\cos x}$	
$=\frac{\sin^2 x+\cos^2 x}{\cos x}$	
$=\frac{1}{\cos x}$	

$\therefore LS=RS$

$\therefore$ QED

e) $\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x$

LS	RS
$=\sin^4 x-\cos^4 x$	$=\sin^2 x-\cos^2 x$
$=\underbrace{(\sin^2 x+\cos^2 x)}_{=1}(\sin^2 x-\cos^2 x)$	
$=\sin^2 x-\cos^2 x$	

$\therefore LS=RS$

$\therefore$ QED

f) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

LS	RS
$= (\sin x - \cos x)^2 + (\sin x + \cos x)^2$	$= 2$
$= \underbrace{\sin^2 x + \cos^2 x}_{=1} - \cancel{2\sin x\cos x}$	
$+ \underbrace{\sin^2 x + \cos^2 x}_{=1} + \cancel{2\sin x\cos x}$	
$= 1 + 1$	
$= 2$	

$\therefore LS = RS$

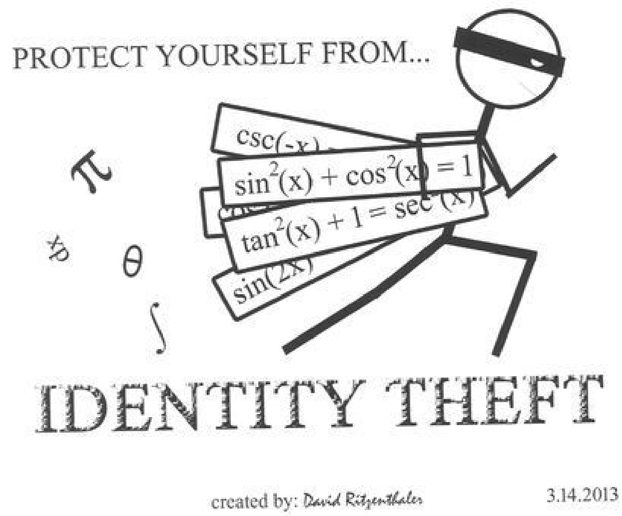
$\therefore$ QED

g) $\dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \dfrac{3 + \sin x}{1 - \sin x}$

LS	RS
$x^2 + 4x + 3 = (x+3)(x+1)$	$= \dfrac{3 + \sin x}{1 - \sin x}$
$= \dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x}$	
$= \dfrac{(\sin x + 3)(\sin x + 1)}{1 - \sin^2 x}$	
$= \dfrac{(\sin x + 3)\cancel{(\sin x + 1)}}{(1 - \sin x)\cancel{(1 + \sin x)}}$	
$= \dfrac{\sin x + 3}{1 - \sin x}$	

$\therefore LS = RS$

$\therefore$ QED


PROTECT YOURSELF FROM...
csc(-x)
$\sin^2(x) + \cos^2(x) = 1$
$\tan^2(x) + 1 = \sec^2(x)$
sin(2x)
π
dx
θ
∫
IDENTITY THEFT
created by: David Ritzenthaler
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