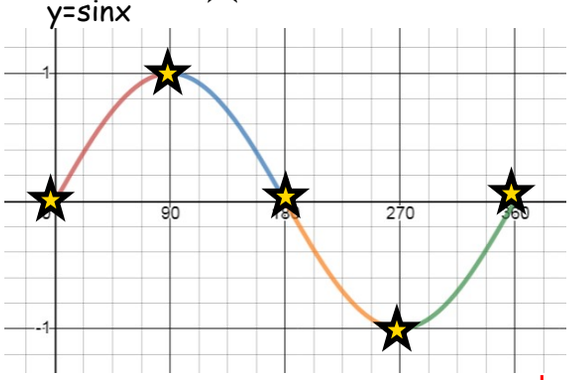
5.3 Stretches of Periodic Functions

To sketch sine and cosine functions, remember the 5 key points: Maximum, Minimum, and zeroes. These 5 points are equally spaced along the x-axis, so they divide the period into quarters.

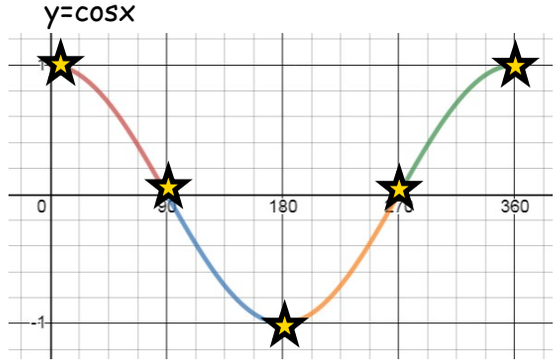
scale = period / 4

Key points

y=sinx



y=cosx



scale = 360° / 4
= 90°

(something happens every 90°)

Explore transformations...

Vertical Compressions and stretches

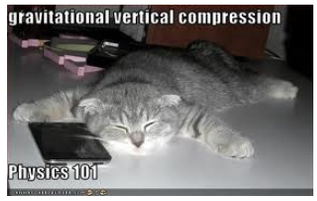
Given y = af(x), when :

$|a| > 1$ vertical stretch by a

$0 < |a| < 1$ vertical compression by $\frac{1}{a}$

$a < 0$ vertical reflection

"a" is called the amplitude



eg: $y = 2\sin x$

Vertical stretch by 2

eg: $y = \frac{1}{3}\cos x$

Vertical compression by 3

Ex 1 Sketch the following for one full cycle.

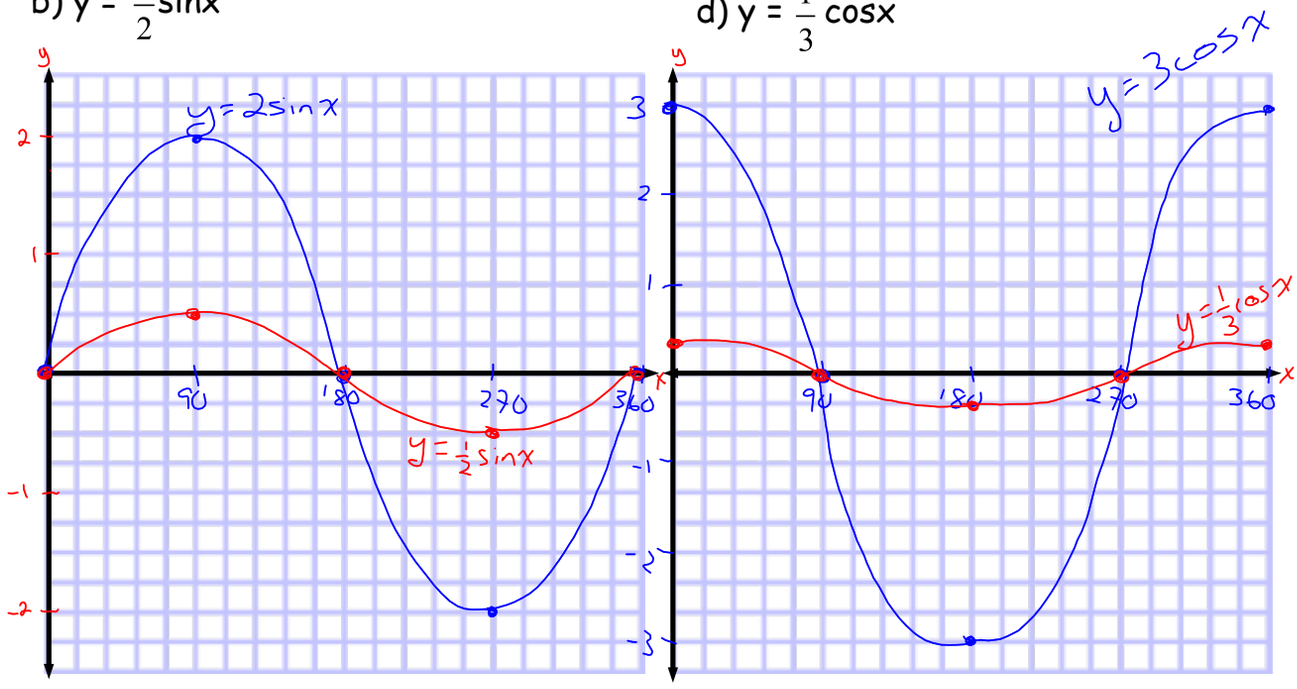
Base $y = \sin x$

a) $y = 2\sin x$ 0, 1, 0, −1, 0

b) $y = \frac{1}{2}\sin x$

c) $y = 3\cos x$ 1, 0, −1, 0, 1

d) $y = \frac{1}{3}\cos x$



For the above examples:
Does the amplitude change?
Does the period change?

Horizontal Stretches or Compressions

Remember, k is INSIDE the function and behaves OPPOSITE from what you would expect.

Given y = f(kx), when:

$|k| > 1$ Horizontal Compression by k

$0 < |k| < 1$ Horizontal stretch by $\frac{1}{k}$

$k < 0$ Horizontal Reflection

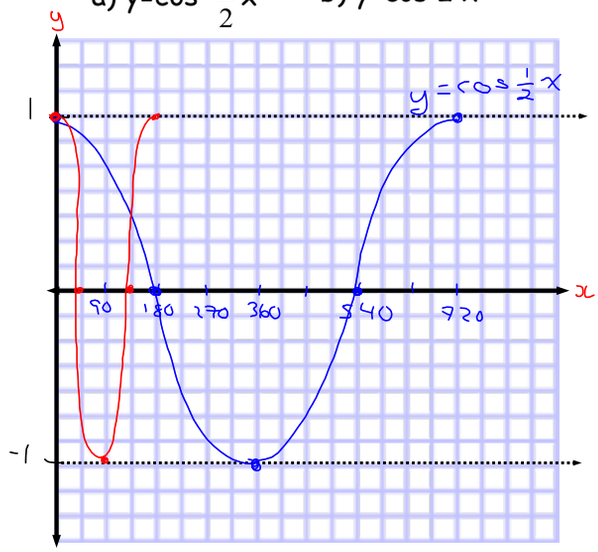
Because you are stretching/compressing horizontally, the period would change.

PERIOD = $\frac{360^0}{k}$

Ex 2 Sketch the following for one full cycle.

1, 0, -1, 0, 1

a) y=cos $\frac{1}{2}x$ b) y=cos 2 x



a) Period = $\frac{360}{\frac{1}{2}}$

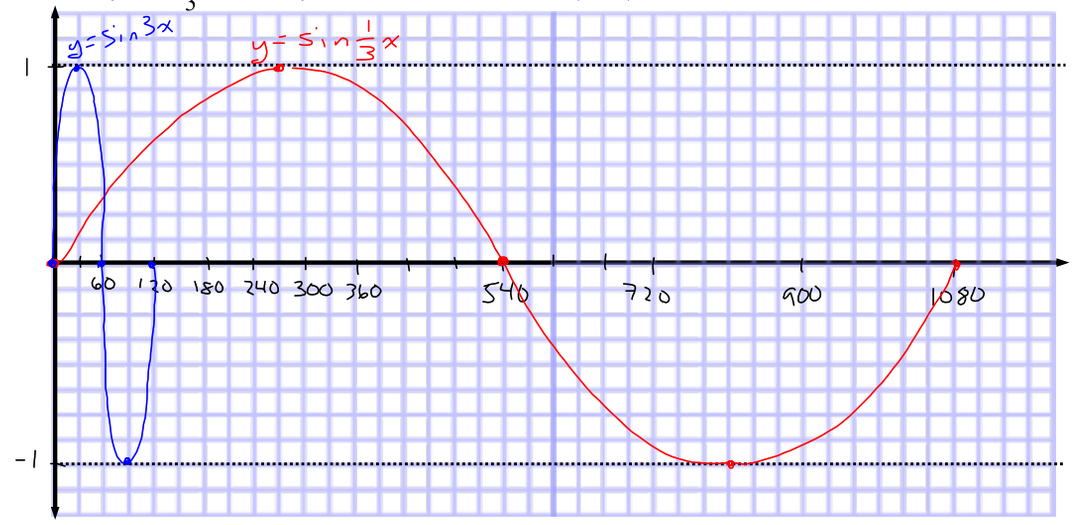
= 720

b) Period = $\frac{360}{2}$

= 180

For these examples:
Does the amplitude change?
Does the period change?

c) y=sin $\frac{1}{3}x$ d) y=sin3x 0, 1, 0, -1, 0



c) $y = \sin\frac{1}{3}x$

Period = $\frac{360}{\frac{1}{3}}$

= 1080

Major Points "scale" = $\frac{1080}{4}$

= 270

d) $y = \sin 3x$

Period = $\frac{360}{3}$

= 120

Scale = $\frac{120}{4}$

= 30

Ex 3 Graph one cycle of $y = 2\sin\left(\frac{1}{2}x\right)$

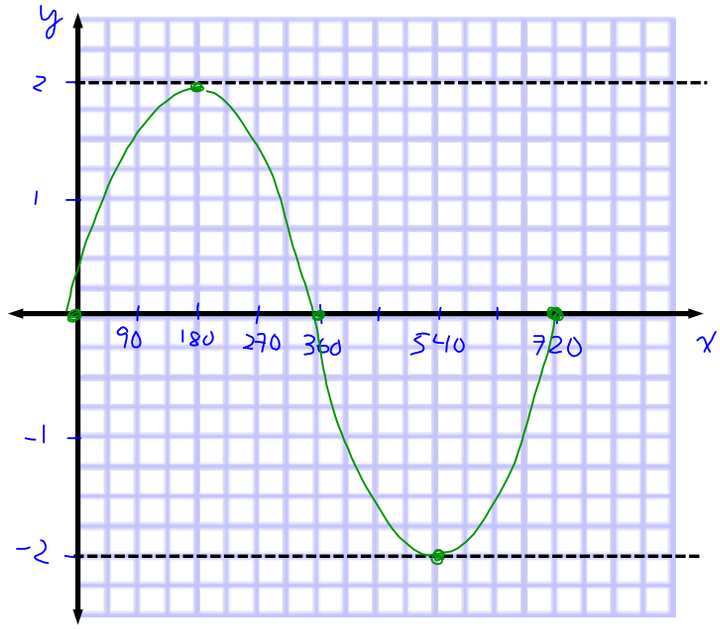
0, 1, 0, −1, 0

Period = $\frac{360}{\frac{1}{2}}$

= 720

Scale = $\frac{720}{4}$

= 180



Max = 2

Min = −2

- Determine: Period, max, min, scale
- Set up your axes
- Draw guides
- Use major points to graph.

Homework

Handout pages

p. 309 # 1ac, 2bd, 3 (and sketch a,c,f,h),

p 312 #18