5.4 -Combination of Transformations

Vertical Stretch/Compression
(AMPLITUDE)

Horizontal Shift
left/right
(PHASE SHIFT)

In general: $y = af(k(x-d))+c$

Horizontal Stretch/Compression

Vertical Shift
up/down
"Function Axis"
(Imaginary Middle)

Perform transformations in the following order:

1) Reflections and Stretches/compressions together
2) Translations

period= $\frac{360^0}{k}$

Don't forget to factor out the coefficient of the x-term!

$\sin(2x-90) = \sin[2(x-45)]$

The amplitude is |a|

$$a = \frac{\max - \min}{2}$$

Max= c + |a|
Min = c - |a|

y=c is the base line/imaginary middle
(axis of the curve)

$$y = \frac{\max + \min}{2}$$

Ex 1 - Describe the transformations from the base function.

a) $y = -2\sin 3x$

① Vertical Reflection (over x-axis)
② Vertical stretch by 2
③ Horizontal Compression by 3

b) $h(x) = 3\sin(x - 60^0) + 2$

① V.S. by 3
② Shift right 60° ("P.S. right 60°")
③ Shift up 2

c) $y = 2\sin(2x + 45^0) + 3$

$= 2\sin[2(x + 22.5^\circ)] + 3$

① V.S. by 2
② H.C. by 2
③ P.S. left 22.5°
④ Shift up 3

d) $y = \frac{-1}{2}\cos(4x - 180^0)$

$= -\frac{1}{2}\cos[4(x - 45^\circ)]$

① Vertical Reflection
② V.C. by 2
③ H.C. by 4
④ P.S. right 45°

Ex 2 - Sketch the graph for each, pay attention to the restrictions. State the amplitude, period, D & R, phase shift and vertical translation.

a) $y = 3\sin\left(\frac{1}{2}x - 30°\right)$ for one cycle

$y = 3\sin\left[\frac{1}{2}(x - 60°)\right]$

P.S. = 60° right

$c = 0$

max = 0 + 3 = 3

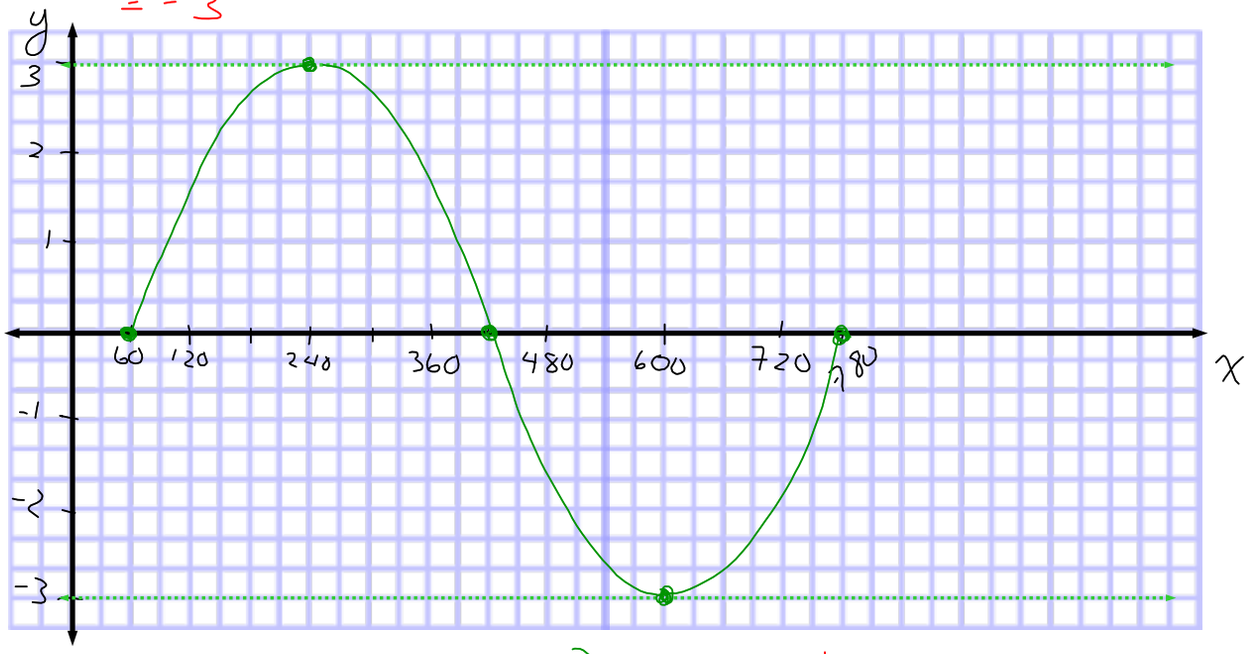
min = 0 − 3 = −3

$a = 3$

period = $\frac{360}{\frac{1}{2}}$ = 720

Scale = $\frac{720}{4}$ = 180

Pattern: 0, 1, 0, −1, 0



D: $\{x \in \mathbb{R} \mid 60 \le x \le 780\}$ or ... $\mid 0 \le x \le 720$

R: $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

b) $y = -2\cos(2x - 90^0) - 2, \quad 0 \le x \le 360^o$

$y = -2\cos[2(x-45^\circ)] - 2$

$a = 2$

$\text{period} = \frac{360}{2} = 180$

$\text{scale} = \frac{180}{4} = 45$

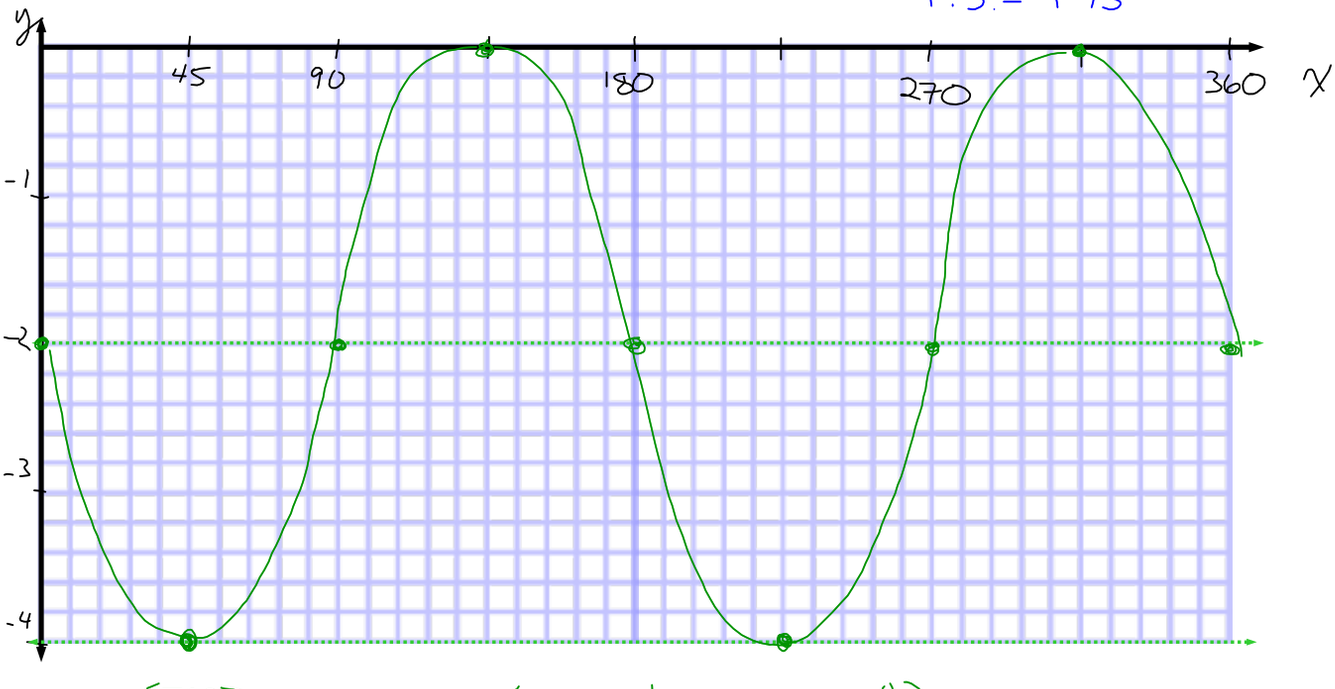
$\text{P.S.} = +45^\circ$

$c = -2$

$\text{max} = -2 + 2 = 0$

$\text{min} = -2 - 2 = -4$

$-\cos : -1, 0, 1, 0, -1$



$D: \{x \in \mathbb{R} \mid 0 \le x \le 360^\circ\}$

$R: \{y \in \mathbb{R} \mid -4 \le y \le 0\}$

Homework
Pg. 383 # 1ab, 2, 5, 7ace
5.4 Handout