5.6B Applications of Trig Functions-continued

Ex 1. The temperature of a solar-heated pool changes throughout a sunny day and is modeled by a trigonometric relation. The temperature ranges from 23^0C at 6 A.M. to 29^0C at 6 P.M.

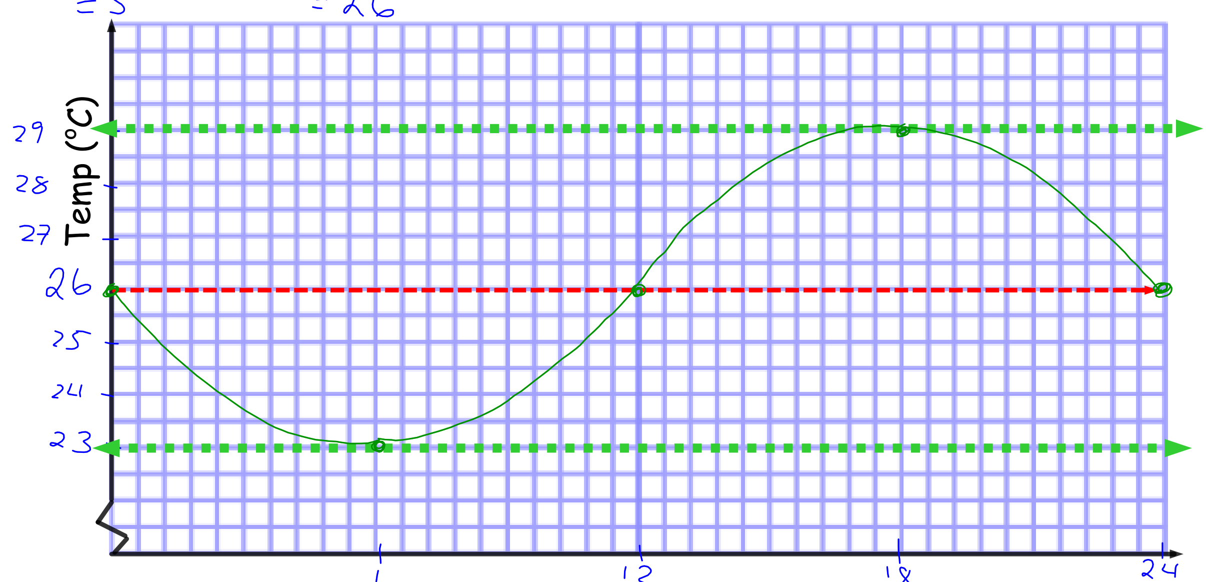
min max

$\frac{1}{2}$ period = 12 hrs

∴ Period = 24 hrs

a) Graph the relation for a 24 hour period starting at midnight (t=0).

$a = \frac{29-23}{2} = 3$ $c = \frac{29+23}{2} = 26$



b) Determine the equation of a sine function for the given graph.

$a=3$ $d=0$ $k=\frac{360}{24} = 15$

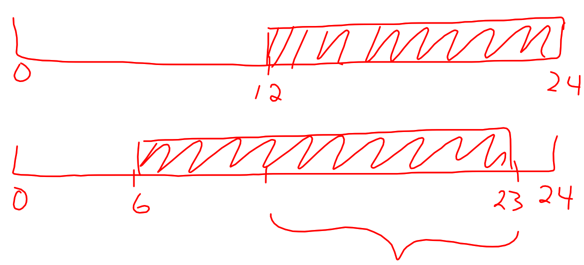
$c=26$

$y = -3\sin(15t)+26$

c) The pool is comfortable for swimming when the temperature is at least 26^0C. The pool is open from 6 A.M. to 11 P.M. every day. How many hours of comfortable swimming are available on a sunny day?

Temp:

Open:

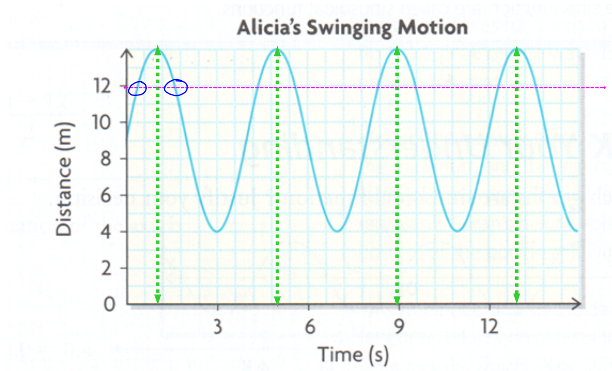


From t=12 to t=23

23−12

=11 hrs per day

Ex 2: Alicia was swinging back and forth in front of a motion detector. Her distance from the detector can be modelled by the equation $d(t)=5\sin 90t + 9$.



a) Find the times when Alicia is 14 m away for the number of swings given. Show graphically and algebraically.

Graphically: Approx: 1, 5, 9, 13s

Algebraically

Solve for when $d = 14$

$14 = 5\sin(90t) + 9$

$5 = 5\sin(90t)$

$1 = \sin(90t)$

$90t = \sin^{-1}(1)$

$90t = 90$

$t = 1$

$14 = 5x + 9$

$5 = 5x$

$1 = x$

$1 = \sin\theta$

$\theta = 90$

Period?

$= \frac{360}{90}$

$= 4$

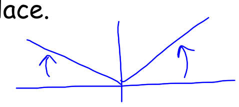
$t_1 = 1s$

$t_2 = 1 + 4s$
$= 5s$

$t_3 = 5 + 4s$
$= 9s$

$\therefore$ 1, 5, 9, 13 s

b) Find the times when Alicia is 12 m away algebraically for the number of swings given. Round to one place.



$12 = 5\sin(90t) + 9$

$\frac{3}{5} = \sin(90t)$

$90t = \sin^{-1}\left(\frac{3}{5}\right)$

$90t \doteq 36.9$

Q_1

$\theta = 36.9$

$90t = 36.9$

$t \doteq 0.4$

Q_2

$\theta = 180 - 36.9$

$= 143.1$

$90t = 143.1$

$t \doteq 1.6$

She is 12m away at $t = 0.4, 1.6, 4.4, 5.6, 8.4, 9.6, 12.4, 13.6$

Ex 3: The depth of the water in a harbour fluctuates because of the tide and is modeled by the equation d = -3cos(30t) + 6 ,
where d represents the depth of the water in metres, and t represents the number of hours after midnight.
(ie. t=0 means midnight, t=3 means 3 A.M. etc)

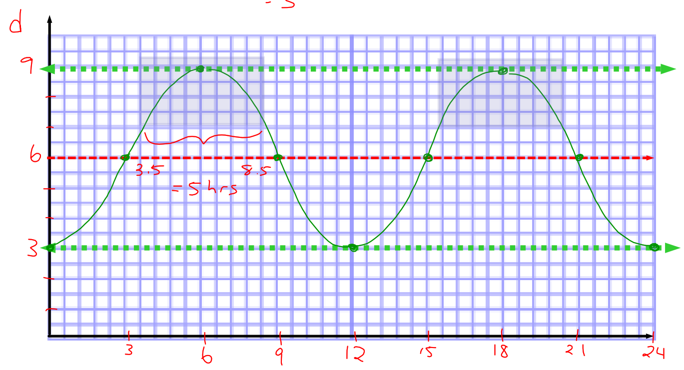
a) Graph the relation for 24 hours

$\text{period} = \frac{360}{30} = 12$

$\text{scale} = \frac{12}{4} = 3$

$\max = 6 + 3 = 9$

$\min = 6 - 3 = 3$



b) Determine the value of d when t=3. Explain what these values represent.

With graph:

d = 6m

∴ at 3am the depth is 6m deep

With equation:

$d(t) = -3\cos(30t) + 6$

$d(3) = -3\cos(30 \cdot 3) + 6$

$= 6$

c) Determine the maximum depth of the water

$a = 3$

$c = 6$

$\therefore \max = 6 + 3 = 9$

∴ Max depth is 9m

d) Surfing is allowed when the depth of the water is 7 metres or more.
Show graphically and algebraically when this occurs?

Shown on graph..
5hrs per cycle,
∴ 10 hrs per day

Algebraically

$7 = -3\cos(30t) + 6$

$-\frac{1}{3} = \cos(30t)$

$30t = \cos^{-1}\left(-\frac{1}{3}\right)$

$\theta_r = 70.5$

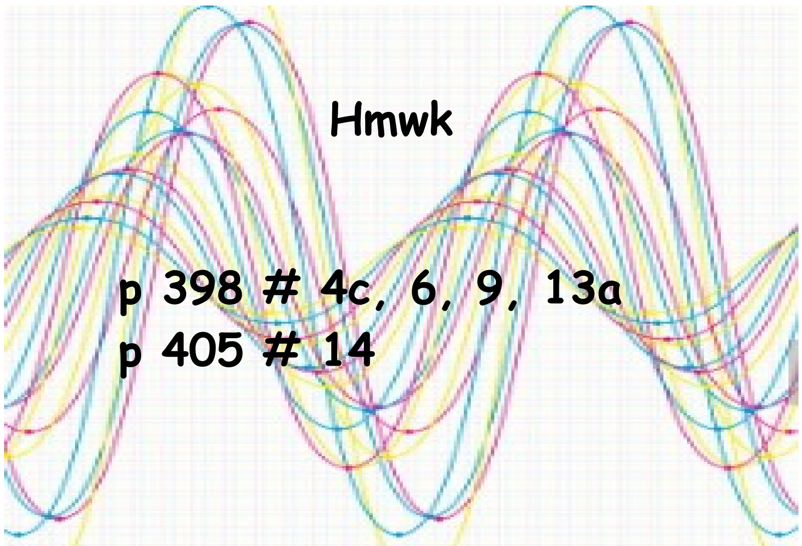
S | A
T | C

Q_2: $30t = 180 - 70.5$, $t = 3.65$

Q_3: $30t = 180 + 70.5$, $t = 8.35$

$\therefore 8.35 - 3.65 = 4.7$ hrs per cycle

9.4 hrs per day



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