6.1 The Equation of a Line

To write the equation of a line you need slope and y-intercept.

Recall: Rate of change = $\frac{\text{rise}}{\text{run}}$

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

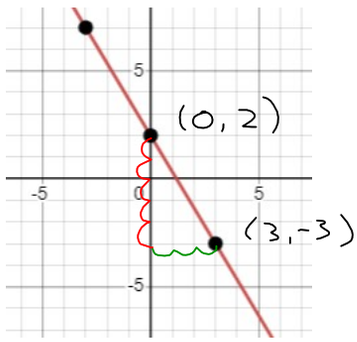
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$y = mx + b$

*slope = rate of change

Ex 1: Determine the slope for each of the following.

a)



$m = \frac{-5}{3}$

OR

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3 - 2}{3 - 0}$

$= -\frac{5}{3}$

b) the line passing through point A(-7,6) and B(21,-10)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

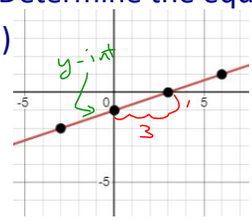
$= \frac{-10 - 6}{21 - (-7)}$

$= \frac{-16^{4}}{28_{7}}$

$= -\frac{4}{7}$

Ex 2: Determine the equation for each of the following lines.

a)

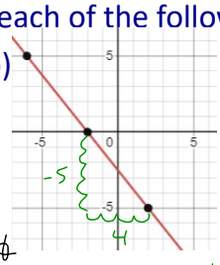


$m = \frac{1}{3}$

$b = -1$

$y = \frac{1}{3}x - 1$

b)



*algebraically determine the value of "b"

$m = \frac{-5}{4}$

Need slope & point

Use any point to solve for b

ex: (-2, 0)

$y = -\frac{5}{4}x + b$

Sub (-2, 0)

$0 = -\frac{5}{4}(-2) + b$

$0 = \frac{5}{2} + b$

$-\frac{5}{2} = b$

$\therefore y = -\frac{5}{4}x - \frac{5}{2}$

c) the line passing through points A(10,3) and B(-4,10)

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - 3}{-4 - 10}$

$= \frac{7}{-14}$

$= -\frac{1}{2}$

② $y = mx + b$

Sub $m = -\frac{1}{2}$ (10, 3)

$3 = -\frac{1}{2}(10) + b$

$3 = -5 + b$

$8 = b$

$\therefore y = -\frac{1}{2}x + 8$

① - Find slope

② - Use equation with slope & a point to solve for b

d) the line passing through points C(3,-4) and D(-1,7)

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7 - (-4)}{-1 - 3}$

$= \frac{11}{-4}$

$= -\frac{11}{4}$

② $y = mx + b$

Sub (-1, 7), $m = -\frac{11}{4}$

$7 = -\frac{11}{4}(-1) + b$

$7 = \frac{11}{4} + b$

$7 - \frac{11}{4} = b$

$\frac{28}{4} - \frac{11}{4} = b$

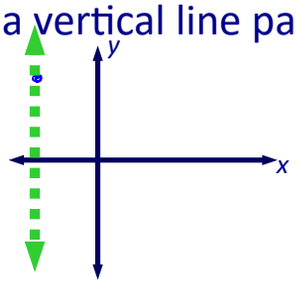
$\frac{17}{4} = b$

$\therefore y = -\frac{11}{4}x + \frac{17}{4}$

SPECIAL CASES: Horizontal & Vertical Lines

Ex 3: Determine the equation for each of the following lines.

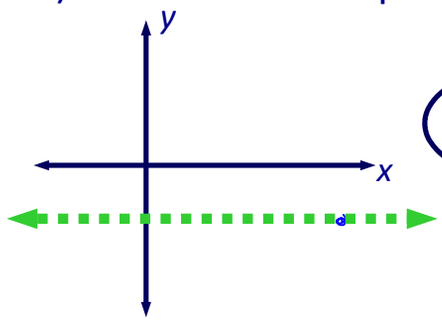
a) a vertical line passing through (-3,5)



the x-coordinate is always -3

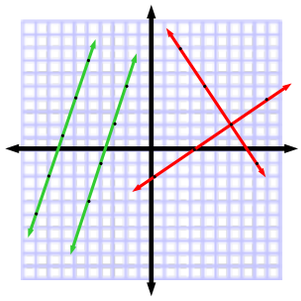
$x = -3$

b) a horizontal line passing through (7,-2)



the y-coordinate is always -2

$y = -2$



Note:

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals.

$m = \frac{1}{3}$ $m_\perp = -\frac{3}{1}$ $m = -\frac{2}{1}$ $m_\perp = \frac{1}{2}$

Ex 4 Determine the equation of a line :

a) parallel to 9x - 3y = 12 and passing through the point D(3,5).

Put into y=mx+b to find slope

$9x - 3y = 12$

$-3y = -9x + 12$

$y = 3x - 4$

Parallel means same slope!

$\therefore m = 3$

Sub in w/ point to solve for b

$y = mx + b$

$5 = 3(3) + b$

$b = -4$

$\therefore y = 3x - 4$

b) perpendicular to y = -2x +7 and passing through the point D(2,-3).

$m = -2$

$m_\perp = \frac{1}{2}$

$y = mx + b$

$-3 = \frac{1}{2}(2) + b$

$-3 = 1 + b$

$b = -4$

$\therefore y = \frac{1}{2}x - 4$

c) perpendicular to 4x +3y - 7 = 0 and has the same x-intercept as 2x + 3y - 12 = 0

Set y=0

- rearrange
- take perp. slope

$2x + 3y - 12 = 0$

Set $y = 0$

$2x + 3(0) - 12 = 0$

$2x = 12$

$x = 6$

$\therefore (6,0)$

$4x + 3y - 7 = 0$

$3y = -4x + 7$

$y = -\frac{4}{3}x + \frac{7}{3}$

$\therefore m = -\frac{4}{3}$

$m_\perp = \frac{3}{4}$

$y = mx + b$

$0 = \frac{3}{4}(6) + b$

$0 = \frac{9}{2} + b$

$-\frac{9}{2} = b$

$\therefore y = \frac{3}{4}x - \frac{9}{2}$