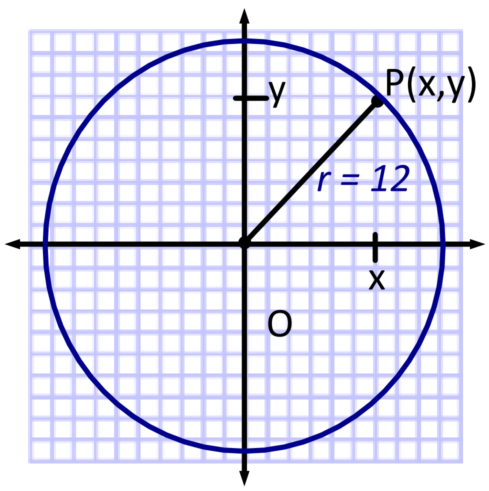
6.4 Equation of a Circle

Consider the general point P(x, y) on the circle whose centre is at the origin (0,0) and whose radius is 12...



What is true for all points on the circle?

The distance from (0, 0) will always equal 12.

Using the distance formula, we can now write the equation of the circle:

$$\sqrt{x^2 + y^2} = 12$$

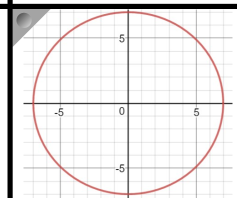
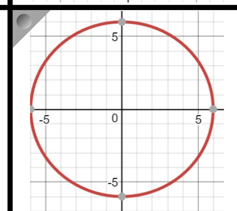
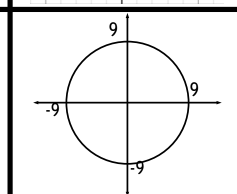
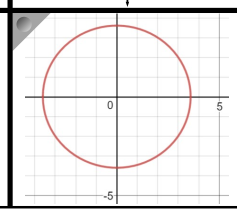
or squaring both sides:

$$x^2 + y^2 = 144$$

In general, the equation of a circle with centre (0,0) and radius, *r*, is given by

$$\boxed{x^2 + y^2 = r^2}$$

Ex. 1: Complete the table.

equation	centre	radius	sketch	x-int, y-int
$x^2 + y^2 = 49$	$(0,0)$	7		x-ints: ± 7 y-ints: ± 7
$x^2 + y^2 = 36$	(0, 0)	6		x-ints: ± 6 y-ints: ± 6
$x^2 + y^2 = 81$	$(0,0)$	9		x-ints: ± 9 y-ints: ± 9
$x^2 + y^2 = 13$	$(0,0)$	$\sqrt{13}$		x-ints: $\pm\sqrt{13}$ y-ints: $\pm\sqrt{13}$

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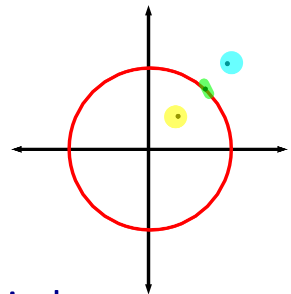
Ex. 2 Consider the circle $x^2 + y^2 = 169$. How could you tell if a given point, $P(x,y)$, is:



a) on the circle
b) inside the circle
c) outside the circle?

a) If $\boldsymbol{x^2 + y^2 = 169}$, then the point is on the circle.
(The point satisfies the equation).

less than b) If $\boldsymbol{x^2 + y^2 < 169}$, then the point is inside the circle.
(The length of line segment PO is shorter than the radius).

greater than c) If $\boldsymbol{x^2 + y^2 > 169}$, then the point is outside the circle.
(The length of line segment PO is longer than the radius).

Ex. 3: Determine whether the following points are on, inside, or outside the circle defined by the equation $\boldsymbol{x^2 + y^2 = 169}$.

a) (-5,12)

$x^2 + y^2 = 169$

LS
$= (-5)^2 + (12)^2$
$= 25 + 144$
$= 169$

RS
$= 169$

LS = RS
∴ Pt. lies ON the circle

b) (11,-4)

LS
$= (11)^2 + (-4)^2$
$= 121 + 16$
$= 137$

RS
$= 169$

LS < RS
∴ Pt. lies INSIDE the circle

c. (10,11)

LS
$= 10^2 + 11^2$
$= 100 + 121$
$= 221$

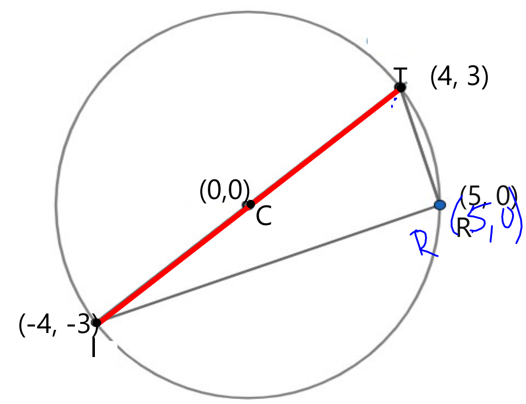
RS
$= 169$

LS > RS
∴ Pt. lies OUTSIDE the circle

Ex. 4: The points T(4,3) and I(-4,-3) are the endpoint of the diameter of a circle with the center at (0,0).

a) Determine the equation of the circle.

b) Point R(5,0) is on the circle, verify that ΔTRI is a right angled triangle.

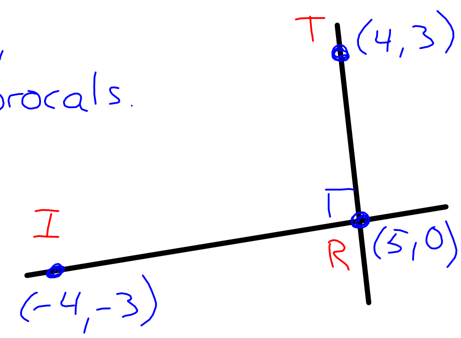


a) Need the radius! Find distance between C and T

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(4-0)^2 + (3-0)^2}$$
$$= \sqrt{16+9}$$
$$= 5$$

$\therefore x^2 + y^2 = 25$

b) If two lines are perpendicular, their slopes are negative reciprocals.



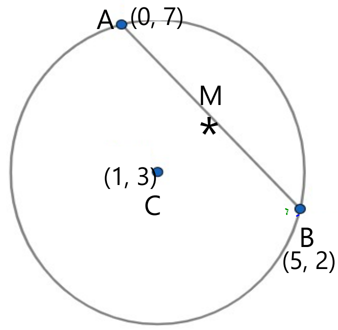
Slopes m_{IR} & m_{TR} should be negative reciprocals

$$m_{IR} = \frac{y_2-y_1}{x_2-x_1} = \frac{0-(-3)}{5-(-4)} = \frac{3}{9} = \frac{1}{3}$$

$$m_{TR} = \frac{0-3}{5-4} = \frac{-3}{1} = -3$$

$\therefore$ Yes! They are perpendicular. This IS a right triangle.

Ex. 4: The points A(0, 7) and B(5, 2) are on the circumference of a circle with center C(1,3), therefore AB is a chord of the circle. Point M is the midpoint of chord AB. Verify algebraically that the line perpendicular to Chord AB and passing through the through M will also pass through C.



Working backwards....

- Goal is to test (1,3) on our line
- ② Need the line! slope between A & B, find perp. slope and build through M.
- ① Need midpoint M.

① Midpoint M

$$M_{AB} = \left(\frac{0+5}{2}, \frac{7+2}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{9}{2}\right)$$

② a) Get slope m_{AB} and use perpendicular

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{\perp} = 1$$
$$= \frac{2-7}{5-0}$$
$$= \frac{-5}{5}$$
$$= -1$$

② b) Build line using $m = 1$ and $\left(\frac{5}{2}, \frac{9}{2}\right)$

$$y = mx + b$$
$$y = 1x + b$$
Sub $\left(\frac{5}{2}, \frac{9}{2}\right)$
$$\frac{9}{2} = 1\left(\frac{5}{2}\right) + b$$
$$\frac{9}{2} - \frac{5}{2} = b$$
$$\frac{4}{2} = b$$
$$2 = b$$

$\therefore y = x + 2$

③ Test (1,3) to confirm is on the line

LS	RS
$= y$	$= x + 2$
Sub (1,3)	
$= 3$	$= 1 + 2$
	$= 3$

$\therefore$ LS = RS

Pt. is on the line!

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