

6.2 - Arithmetic Sequences

A sequence where there is a common difference, d , between consecutive terms. The same value is added or subtracted to a term to generate the next term.

$$\begin{array}{ll} \text{eg. } 3, 5, 7, 9, 11, \dots & d = 2 \\ 5, 1, -3, -7, \dots & d = -4 \\ 0, 5, 10, 15, 20, \dots & d = 5 \end{array}$$

Notice the pattern:

$$1, \quad 4, \quad 7, \quad 10, \quad 13$$

$$1, 1 + 1(3), 1 + 2(3), 1 + 3(3), 1 + 4(3)$$

$$a, a + 1d, a + 2d, a + 3d, a + 4d$$

$a = 1$ $d = 3$
First term
Difference

always 1 less than term #

Arithmetic Sequence Formula

$$t_n = a + (n - 1)d$$

where a is the first term and d is the common difference

Ex. 1 Determine t_n for each.

This means find the general formula which works to find any term in the sequence.
Must be simplified.

a) 7, 3, -1, -5, ...

$$a = 7 \quad t_n = a + (n-1)d$$

$$d = -4 \quad = 7 + (n-1)(-4)$$

$$= 7 - 4n + 4$$

$$t_n = 11 - 4n$$

b) -5, -3, -1, 1, ...

$$a = -5 \quad t_n = a + (n-1)d$$

$$d = 2 \quad = -5 + (n-1)(2)$$

$$= -5 + 2n - 2$$

$$t_n = 2n - 7$$

Ex. 2 Determine the # of terms in each sequence.

① a) 2, 5, 8, ..., 155

$$a = 2 \quad t_n = a + (n-1)d$$

$$d = 3 \quad = 2 + (n-1)(3)$$

$$= 2 + 3n - 3$$

$$t_n = 3n - 1$$

② Use 155 to solve for n

$$155 = 3n - 1$$

$$\frac{156}{3} = n \quad \therefore 52 \text{ terms}$$

$$n = 52$$

b) 1, -1, -3, ..., -199

$$a = 1 \quad t_n = a + (n-1)d$$

$$d = -2 \quad = 1 + (n-1)(-2)$$

$$= 1 - 2n + 2$$

$$t_n = 3 - 2n$$

$$-199 = 3 - 2n$$

$$-202 = -2n$$

$$n = 101$$

$$\therefore 101 \text{ terms}$$

Ex. 3 Insert two numbers between 17 and 59, so that the four numbers form an arithmetic sequence.

$$17 \quad \underline{31} \quad \underline{45} \quad 59$$

\curvearrowright \curvearrowright \curvearrowright
 $+d$ $+d$ $+d$

$$59 = 17 + 3d$$

$$42 = 3d$$

$$14 = d$$

$$t_2 = 17 + 14$$

$$= 31$$

$$t_3 = 31 + 14$$

$$= 45$$

4. Determine, a , d , and t_n for each arithmetic sequence.

a) $t_4 = 13$, $t_{17} = 39$

13 differences
(17-4)

$$t_4 + 13d = t_{17}$$

$$13 + 13d = 39$$

$$13d = 26$$

$$\boxed{d = 2}$$

Need a

Use one term to solve for it

$$t_n = a + (n-1)d$$

$$t_4 = a + (4-1)d$$

$$13 = a + (3)(2)$$

$$\boxed{7 = a}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 7 + (n-1)(2) \\ &= 7 + 2n - 2 \end{aligned}$$

$$\boxed{t_n = 2n + 5}$$

b) $t_{10} = -67$, $t_{43} = -298$

33d

$$t_{10} + 33d = t_{43}$$

$$-67 + 33d = -298$$

$$\boxed{d = -7}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= -4 + (n-1)(-7) \\ &= -4 - 7n + 7 \end{aligned}$$

$$\boxed{t_n = 3 - 7n}$$

$$t_n = a + (n-1)d$$

$$t_{10} = a + (10-1)(-7)$$

$$-67 = a + (9)(-7)$$

$$\boxed{a = -4}$$

p. 424 #1, 3, 6, 8, 10, 11, 13, 15

